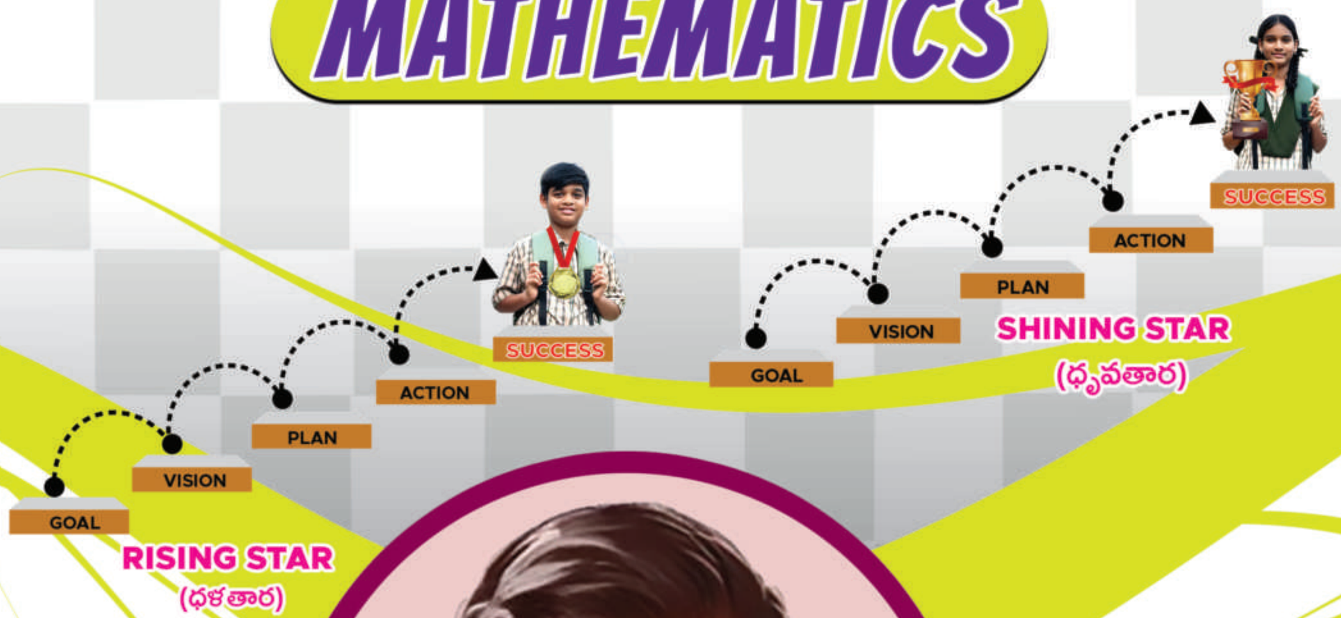
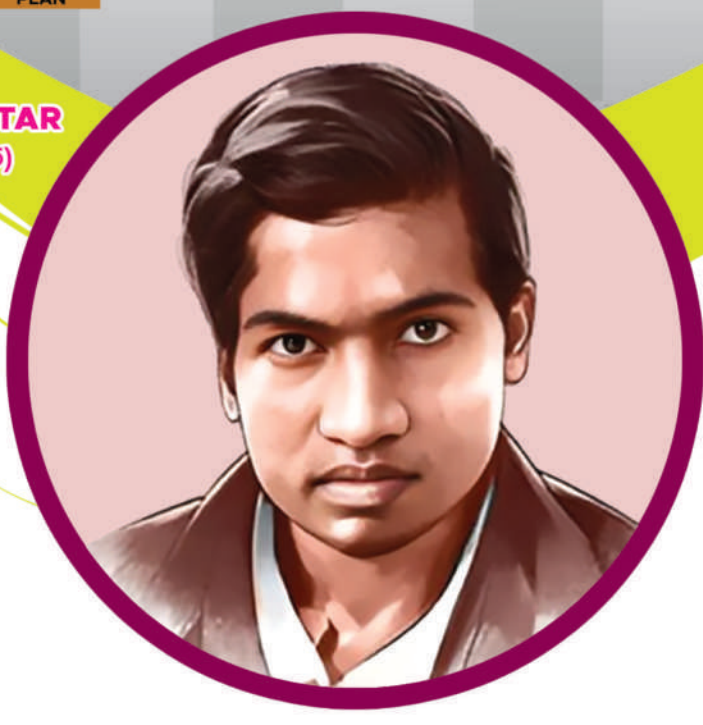
समग्र शिक्षा
Samagra Shiksha
సమగ్ర శిక్షా

# 100 Days Action Plan
## SSC - 2026

# MATHEMATICS



**BOARD OF SECONDARY EDUCATION**

Department of School Education

# 1.REAL NUMBERS

**[ Marks] [1 + 8 = 9 M]**

**LEVEL-1: RISING STAR**

## 8 MARK QUESTION ANSWERS

1. Is $\sqrt{5}$ Irrational ? Justify your Answer.

Solution: Let us assume, to the contrary, that $\sqrt{5}$ is a rational. Then, there exist positive integers a, b such that $\sqrt{5} = \frac{a}{b}$ where a and b are co − prime i.e. their HCF is 1

$\Rightarrow (\sqrt{5})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 5 = \frac{a^2}{b^2} \Rightarrow 5b^2 = a^2$

Therefore, 5 divides $a^2$, then 5 divides a

($\because$ from theorem If p divides $a^2$, then p divides a where a is a positive integer and p be a prime)

So, we can write $a = 5c$ for some integer c.

$\Rightarrow a^2 = 25c^2 \Rightarrow 5b^2 = 25c^2$ ( $\because a^2 = 5b^2$ )

$\Rightarrow b^2 = 5c^2$ This means that, 5 divides $b^2$, then 5 divides b .

Therefore,a and b have at least 5 as a common factor.

But this contradicts the fact that a and b are co − prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational.

So, we conclude that $\sqrt{5}$ is irrational.

2. Is $\sqrt{2}$ Irrational ? Justify your Answer.

Solution: Let us assume, to the contrary, that $\sqrt{2}$ is a rational. Then, there exist positive integers a, b such that $\sqrt{2} = \frac{a}{b}$ where a and b are co − prime i.e. their HCF is 1

$\Rightarrow (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2$

Therefore, 2 divides $a^2$, then 2 divides a

($\because$ from theorem If p divides $a^2$, then p divides a where a is a positive integer and p be a prime)

So, we can write $a = 2c$ for some integer c.

$\Rightarrow a^2 = 4c^2 \Rightarrow 2b^2 = 4c^2$ ( $\because a^2 = 2b^2$ )

$\Rightarrow b^2 = 2c^2$ This means that, 2 divides $b^2$, then 2 divides b

Therefore,a and b have at least 2 as a common factor.

But this contradicts the fact that a and b are co − prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is a rational.

So, we conclude that $\sqrt{2}$ is irrational.

3. Prove that $\sqrt{3}$ is irrational.

Solution: Let us assume, to the contrary, that $\sqrt{3}$ is a rational. Then, there exist positive integers a, b such that $\sqrt{3} = \frac{a}{b}$ where a and b are co − prime i.e. their HCF is 1

$\Rightarrow (\sqrt{3})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 3 = \frac{a^2}{b^2} \Rightarrow 3b^2 = a^2 \Rightarrow b^2 = \frac{a^2}{3}$

Therefore, 3 divides $a^2$, then 3 divides a

($\because$ from theorem If p divides $a^2$, then p divides a where a is a positive integer and p be a prime)

So, we can write $a = 3c$ for some integer c.

$\Rightarrow a^2 = 9c^2 \Rightarrow 3b^2 = 9c^2$ ( $\because a^2 = 3b^2$ )

$\Rightarrow b^2 = 3c^2$ This means that, 3 divides $b^2$, then 3 divides b .

Therefore,a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are co – prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is a rational.

4. Prove that $\sqrt{7}$ is irrational.

Solution: Let us assume, to the contrary, that $\sqrt{7}$ is a rational.

Then, there exist positive integers a, b such that $\sqrt{7} = \frac{a}{b}$ where a and b are co – prime i. e. their HCF is 1

$\Rightarrow (\sqrt{7})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2$ Therefore, 7 divides $a^2$, then 7 divides a

($\because$ from theorem If p divides $a^2$, then p divides a where a is a positive integer and p be a prime)
So, we can write $a = 7c$ for some integer c.

$\Rightarrow a^2 = 49c^2 \Rightarrow 7b^2 = 49c^2$ ( $\because a^2 = 7b^2$ )

$\Rightarrow b^2 = 7c^2$ This means that, 7 divides $b^2$, then 7 divides b .

Therefore,a and b have at least 7 as a common factor.

But this contradicts the fact that a and b are co – prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{7}$ is a rational.

## LEVEL-2: SHINING STAR

### 8 MARK QUESTION ANSWERS

1.Examine whether $3+2\sqrt{5}$? Justify your Answer.

Solution: Let us assume on the contrary that $3+2\sqrt{5}$ is rational.

Then, there exist co- prime positive integers a, b such that

$$3 + 2\sqrt{5} = \frac{a}{b} \Rightarrow 2\sqrt{5} = \frac{a}{b} - 3 \Rightarrow 2\sqrt{5} = \frac{a-3b}{b}$$

$\Rightarrow \sqrt{5} = \frac{a-3b}{2b} \Rightarrow \sqrt{5}$ is rational [ $\because$ a, b are integers , $\therefore \frac{a-3b}{2b}$ is a rational ]

this contradicts the fact that $\sqrt{5}$ is irrational . so our assumption is incorrect.

So, we conclude that $3+2\sqrt{5}$ is an irrational.

2.Is $5-\sqrt{3}$ is Irrational? Justify your Answer.

Solution: Let us assume on the contrary that $5-\sqrt{3}$ is rational.

Then, there exist co- prime positive integers a, b such that $5-\sqrt{3} = \frac{a}{b} \Rightarrow 5 - \frac{a}{b} = \sqrt{3}$

$\Rightarrow \frac{5b-a}{b} = \sqrt{3} \Rightarrow \sqrt{3}$ is rational [ $\because$ a, b are integers, $\therefore \frac{5b-a}{b}$ is a rational ]

But this contradicts the fact that $\sqrt{3}$ is irrational . so our assumption is incorrect.

So, we conclude that $5 - \sqrt{3}$ is Irrational

3.Examine whether $6 + \sqrt{2}$ is Irrational or not ?

Solution: Let us assume on the contrary that $6 + \sqrt{2}$ is rational.

Then, there exist co- prime positive integers a, b such that

$6 + \sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{b} - 6 \Rightarrow \sqrt{2} = \frac{a-6b}{b} \Rightarrow \sqrt{2}$ is rational [ $\because$ a, b are integers , $\frac{a-6b}{b}$ is a rational ] but this contradicts the fact that $\sqrt{2}$ is irrational . so our assumption is incorrect.

So, we conclude that $6 + \sqrt{2}$ is an irrational.

4.Give an example for an irrational number. Also prove that it is an Irrational.

Example: $\sqrt{2}$ Irrational .

Solution: Let us assume, to the contrary, that $\sqrt{2}$ is a rational. Then, there exist positive integers a, b such that $\sqrt{2} = \frac{a}{b}$ where a and b are co − prime i. e. their HCF is 1

$\Rightarrow (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow 2b^2 = a^2$ Therefore, 2 divides $a^2$, then 2 divides a

($\because$ from theorem If p divides $a^2$, then p divides a where a is a positive integer and p be a prime)
So, we can write $a = 2c$ for some integer c.

$\Rightarrow a^2 = 4c^2 \Rightarrow 2b^2 = 4c^2 \; (\because a^2 = 2b^2)$

$\Rightarrow b^2 = 2c^2$ This means that, 2 divides $b^2$, then 2 divides b

Therefore,a and b have at least 2 as a common factor.

But this contradicts the fact that a and b are co − prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is a rational.

So, we conclude that $\sqrt{2}$ is irrational.

**1 MARK QUESTION ANSWERS**

1.Explain the fundamental theorem of arithmetic.

Every composite number can be expressed (factorised ) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

2.HCF (20, 35) = 5 then LCM (20, 35) = ..................... A)140 B) 75 C) 250 D) 350 (A)

3.LCM (a, b) = a × b (a b) then HCF (a, b) = .............. A) ab B) a C) b D) 1 (D)

4.HCF (a, 20) = 2, LCM (a, 20) = 60 then a = ......... A)2 B) 6 C) 12 D) 24 (B)

5.Unit digit in the Product of $6^n$, (n is natural number) A) 9 B) 3 C) 6 D) 2 (C)

6.A, B are positive integers $A = p^3q^2$, $B = p\,q^3$ (p, q are prime numbers) then HCF (A, B) =.............

A) $p\,q^2$ B) $p^2q^2$ C) $p^3q^3$ D) 1 ( A )

7.If n is any natural number, then which the following expression ends with zero (C)

A) $(3 \times 2)^n$ B) $(4 \times 3)^n$ C) $(2 \times 5)^n$ D) $(6 \times 2)^n$

8.Express 153 as product of prime factors ..............

$156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$.

9. Classify $\sqrt{3}$, $\sqrt{4}$, $\pi$ into rational and irrational numbers?

Rational numbers : $\sqrt{4} = 2$

Irrational numbers : $\sqrt{3}$ , , $\pi$

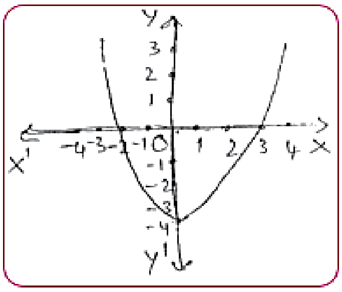
10. If P is a prime number of P divides $a^2$ then P also divides a

# 2.POLYNOMIALS

**[8 Marks] [1 + 1 + 2 + 4 = 8 M]**

**LEVEL-1: RISING STAR**

## 4 MARKS QUESTION ANSWERS



1.By observing the graph answer the following questions.

a. What is the shape of graph in the figure?

The shape of graph in the figure is parabola

b. How many zeroes are there for that polynomial?

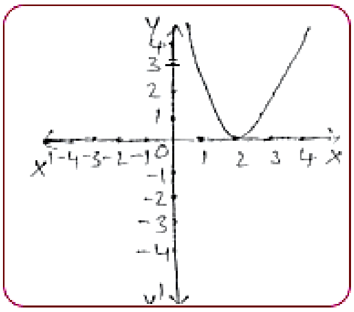
There are two zeroes for that polynomial.

c. Write the zeroes of the polynomial.

The zeroes of the polynomial are $-2, 3$.

d. Find the product of zeroes of the polynomial.

The product of zeroes of the polynomial $= -2 \times 3 = -6$



2.By observing the graph answer the following questions.

a. What is the name of the polynomial in the graph?

The name of the polynomial in the graph is Quadratic polynomial.

b. How many zeroes it has?

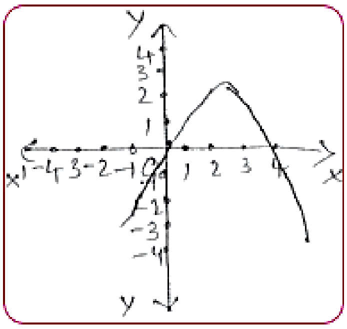
It has one zero

c. Write the zeroes of polynomial

The zeroes of polynomial are 2,2 (Zero with multiplicity 2)

d. Find the sum of zeroes of polynomial.

The sum of zeroes of polynomial $= 2 + 2 = 4$



3. By observing the graph answer the following questions.

a. What is the shape of graph in the figure?

The shape of graph in the figure is parabola

b. writes the zeroes of the polynomial.

The zeroes of the polynomial are 0 and 4 .

c. Find the sum of zeroes of the polynomial.

The sum of zeroes of polynomial $= 0 + 4 = 4$

d. Find the product of zeroes of polynomial

The product of zeroes of the polynomial $= 0 \times 4 = 0$

## 2 MARKS QUESTION ANSWERS

1) Generate a quadratic polynomial whose sum and product of zeroes are $\frac{1}{4}, 1$

Solution: Given sum of its zeroes $= \frac{1}{4}$ and product of its zeroes $= 1$

We know that the general form of a quadratic polynomial is:

k ( $x^2$ − ( sum of roots ) x + ( product of roots )

$= k\,(x^2 - \left(\frac{1}{4}\right)x + (1)) = k\,(x^2 - \left(\frac{1}{4}\right)x + 1)$

We take k= 4 then the quadratic polynomial $= 4\,(x^2 - \left(\frac{1}{4}\right)x + 1) = 4 \times x^2 - 4 \times \left(\frac{1}{4}\right)x + 4 \times 1)$

$= x^2 - 4x + 1$

2) Create a quadratic polynomial whose sum and product of a zeroes are $\sqrt{3}$ and $-\sqrt{3}$

Solution: Given sum of its zeroes $= \sqrt{3}$ and product of its zeroes $= -\sqrt{3}$

We know that the general form of a quadratic polynomial is:

k ( $x^2$ − (sum of roots ) $x$ + ( product of roots )

$= k\,(x^2 - (\sqrt{3})x + (-\sqrt{3})) = k\,(x^2 - (\sqrt{3})x + (-\sqrt{3}))$

We take k= 1 then the quadratic polynomial $= 1\,(x^2 - \sqrt{3}\,x - \sqrt{3}) = x^2 - \sqrt{3}x - \sqrt{3}$

## LEVEL-2: SHINING STAR

### 4 MARKS QUESTION ANSWERS

1.Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between zeroes and coefficients.

Solution: $x^2 - 3 = (x + \sqrt{3})(x - \sqrt{3})$ [ $\because$ from algebraic identity $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow (x + \sqrt{3}) = 0 \Rightarrow x = -\sqrt{3}$ or $(x - \sqrt{3}) = 0 \Rightarrow x = \sqrt{3}$

the zeroes of the polynomial $x^2 - 3$ are $-\sqrt{3}$ and $\sqrt{3}$

verify the relationship between zeroes and coefficients:

sum of zeroes = $-\sqrt{3} + \sqrt{3} = 0 = \frac{-\text{ coefficient of } x}{\text{coefficient of } x^2}$

product of zeroes = $-\sqrt{3} \times \sqrt{3} = -3 = \frac{-3}{1} = \frac{\text{constant}}{\text{coefficient of } x^2}$

$\therefore$ The relationship is verified.

$x^2 - 3$ here
coefficient of $x = 0$
coefficient of $x^2 = 1$
constant $= -3$

### 2 MARKS QUESTION ANSWERS

1.Find the zeroes of quadratic polynomial $P(x) = x^2 - 2x - 8$

Solution: Given quadratic polynomial $P(x) = x^2 - 2x - 8$

$P(x) = 0 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x - 4) + 2(x - 4) = 0 \Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow (x - 4) = 0 \Rightarrow x = 4$ *or* $(x + 2) = 0 \Rightarrow x = -2$

$\therefore -2$ and $4$ are the zeroes of quadratic polynomial .

2.Find the sum and product of zeroes of the polynomial $P(x) = 4x^2 + 8x$

Solution: Given quadratic polynomial $P(x) = 4x^2 + 8x$

This is in the standard quadratic form $ax^2 + bx + c$

Comparing the coefficients, we find that: $a = 4, \; b = 8, \; c = 0$

The formula for the sum of zeroes $(\alpha + \beta) = \frac{-b}{a} = \frac{-8}{4} = 2$

The formula for the product of zeroes $(\alpha\beta = \frac{c}{a} = \frac{0}{4} = 0$

The sum of the zeroes $= -2$ and the product of the zeroes $= 0$.

### 1 MARK QUESTION ANSWERS

1. The graph of the polynomial $P(x) = 3x - 1$ meets x-axis at the point ( $\frac{1}{3}, 0$ )

[ $P(x) = 3x - 1 = 0 \Rightarrow 3x - 1 = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$ ]

2.Assertion (A): The polynomial $P(x) = 4x + 3$ is a linear polynomial ( )

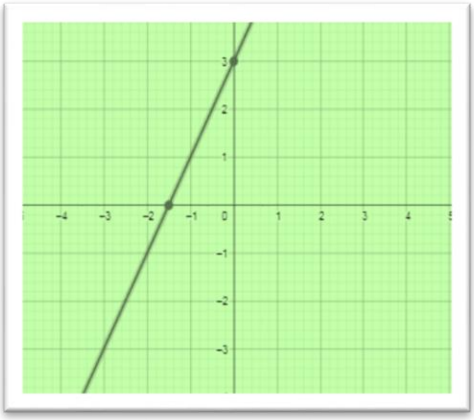
Reason (R): The general form of linear polynomial is $ax + b$

Which of the following is true?

A) Both A, R are true ✓
B) Both A, R are false
C) A is true, R is false
D) A is false, R is true

3.The degree of the polynomial of P(x) = $5x^3 - 4x^2 + x - \sqrt{2}$ is 3

4. Draw the graph of P ($x$) = $2x + 3$.



If $x$ =0, P(0) = $2\times 0 + 3 = 0 + 3 = 3$. The point is (0, 3)

If $x = -1$, P $(-1) = 2\times(-1) + 3 = -2 + 3 = 1$. The point is (-1, 1)

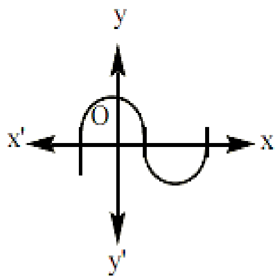
$P(x) = 2x + 3 = 0 \Rightarrow 2x + 3 = 0 \Rightarrow 2x = -3$

$\Rightarrow x = \frac{-3}{2} = -1.5$ The point is ( – 1.5 ,0)

5.The zeroes of $x^2 - 4$ are 2 and – 2 (True/False)

6. The coefficient of $x^2$ in P ($x$) = $7x^3 - 6x^2 + 5x + 8$ is 6

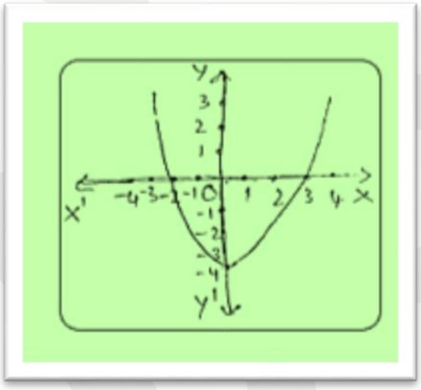
7. The graph of polynomial is shown in the figure then the number of zeroes are 3




8. Product of zeroes of $2x^2 + 6x + m$ is – 1 then $m$ = $-2$

[product of zeroes $(\alpha\beta) = \frac{c}{a} = \frac{m}{2} = -1 \Rightarrow m = 2 \times -1 = -2$]

9. Draw the rough diagram of quadratic polynomial.



10. Write a polynomial whose zeroes are 2 and 4. $x^2 - 6x + 8$

A quadratic polynomial in terms of the zeroes (α, β)is given by
k[ $x^2$ – (sum of the zeroes)x + (product of the zeroes)]
$p(x) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$
Now, Given that zeroes of a quadratic polynomial are 2 and 4
let $\alpha = 2$ and $\beta = 4$
Therefore, substituting the value $\alpha = 2$ and $\beta = 4$ we get
$p(x) = k[x^2 - (2+4)x + 2\times 4] = k[x^2 - 6x + 8]$
if k= 1 then polynomial: $p(x) = x^2 - 6x + 8$

# 3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

**[9 Marks] [1 + 8 = 9 M]**

**LEVEL-1: RISING STAR**

## 8 MARKS QUESTIONS ANSWERS

1.Solve the following pair of linear equation graphically $x - y = 4$ ; $x - 2y = 6$.

Solution: The given pair of linear equations:

$x - y = 4$ ----------(i)   $x - 2y = 6$----------(ii)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

We give these solutions in table shown below

for eq ( i ): $x - y = 4 \Rightarrow y = x - 4$

If $x = 0$ , $y = x - 4 \Rightarrow y = 0 - 4 \Rightarrow y = -4$; point$(0, -4$

If $x = 4$ , $y = x - 4 \Rightarrow y = 4 - 4 \Rightarrow y = 0$; point$(4,0)$

| $x$ | 0 | 4 |
|---|---|---|
| y | $-4$ | 0 |
| point | $(0,-4)$ | $(4, 0)$ |

Plotting the points A$(0, -4)$and B $(4,0)$

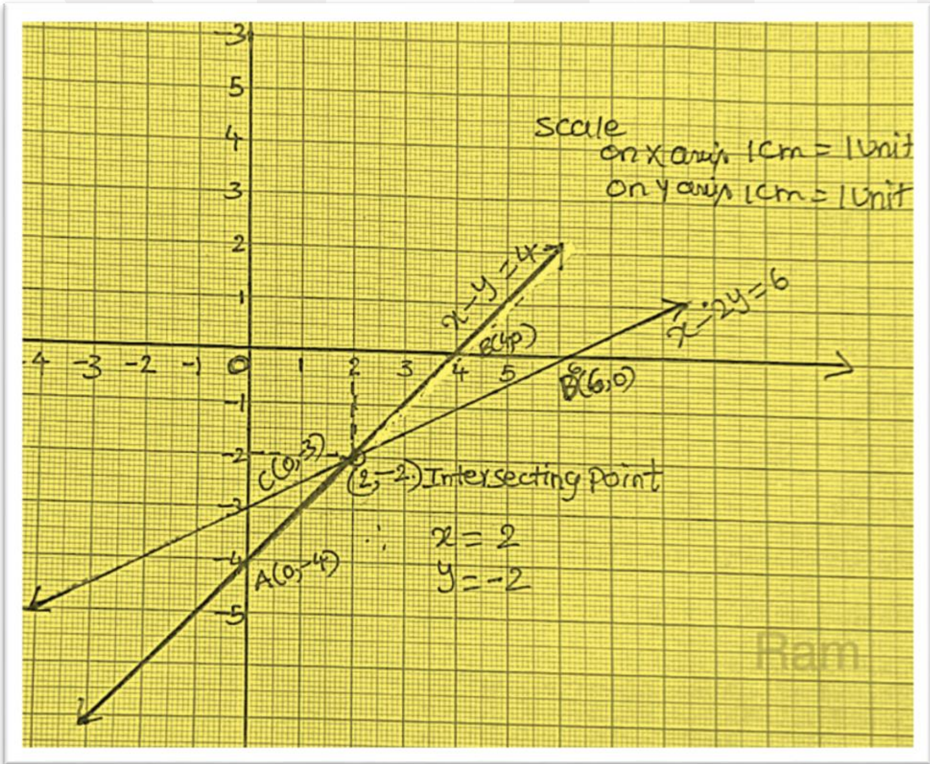
For eq (ii): $x - 2y = 6 \Rightarrow$ y = $\frac{x-6}{2}$

If $x = 0$ , y = $\frac{x-6}{2} \Rightarrow$ y = $\frac{0-6}{2} \Rightarrow$ y = $\frac{-6}{2} \Rightarrow y = -3$ ; point $(0, -3)$

If $x = 6$ , y = $\frac{x-6}{2} \Rightarrow$ y = $\frac{6-6}{2} \Rightarrow$ y = $\frac{0}{2} \Rightarrow y = 0$ ; point $(6,0)$

| $x$ | 0 | 6 |
|---|---|---|
| y | $-3$ | 0 |
| point | $(0,-3)$ | $(6, 0)$ |

Plotting the points C$(0, -3)$and D$(6,0)$




We plot the points on graph. from the graph the points intersect at the point $(x, y) = (2,2)$

$\therefore x = 2$ and $y = 2$

2.Solve the following pair of linear equation graphically $2x + 3y - 6 = 0 - 6; x + y + 3 = 0$

Solution: The given pair of linear equations:

$2x + 3y - 6 = 0 \Longrightarrow 2x + 3y = 6$ ----------(i)

$x + y + 3 = 0 \Longrightarrow x + y = -3$ ----------(ii)

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

We give these solutions in table shown below

for eq (i ): $2x + 3y = 6 \Longrightarrow y = \frac{6-2x}{3}$

If $x = 0, y = \frac{6-2x}{3} \Longrightarrow y = \frac{6-2\times 0}{3} \Longrightarrow y = \frac{6-0}{3} \Longrightarrow y = \frac{6}{3} \Longrightarrow y = 2$; point $(0, 2)$

If $x = 3, y = \frac{6-2x}{3} \Longrightarrow y = \frac{6-2\times 3}{3} \Longrightarrow y = \frac{6-6}{3} \Longrightarrow y = \frac{0}{3} \Longrightarrow y = 0$; point $(3, 0)$

| $x$ | 0 | 3 |
|---|---|---|
| y | 2 | 0 |
| point | (0,2) | (3,0) |

Plotting the points A (0,2)and B (3, 0)

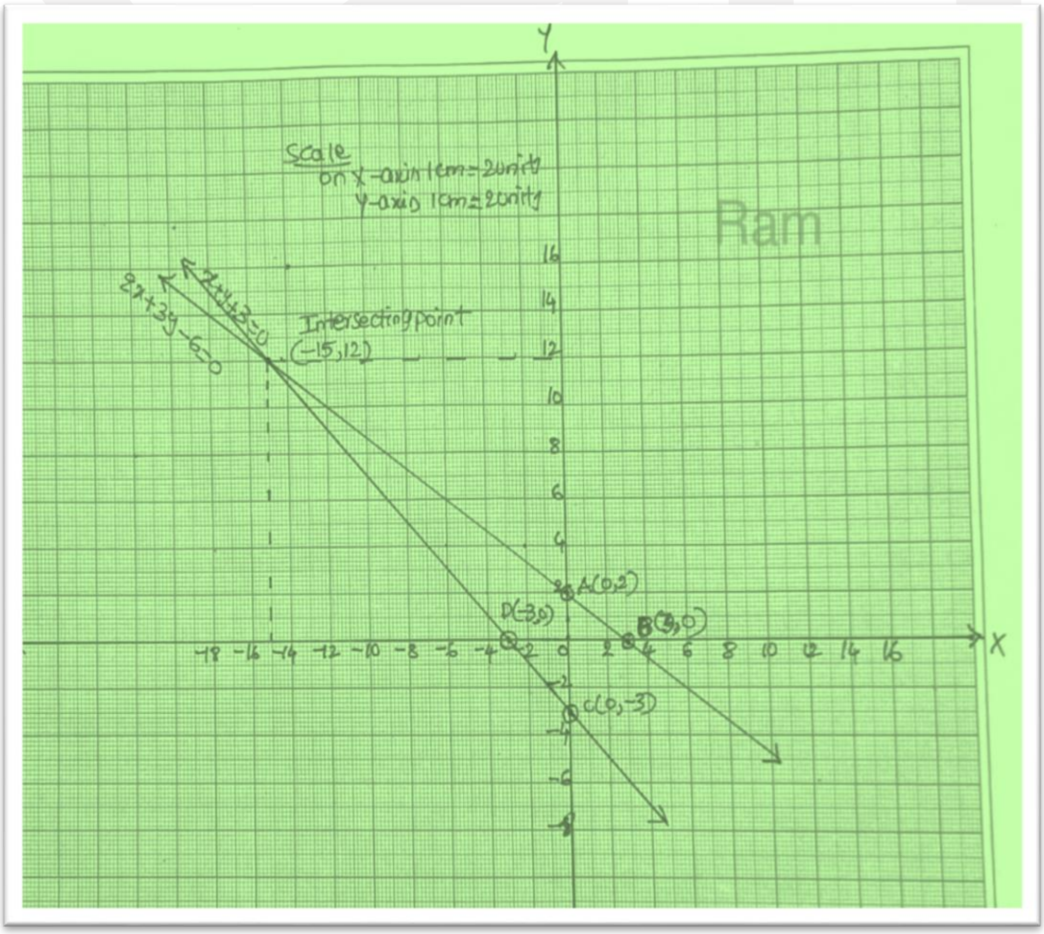
for eq (ii ): $x + y = -3 \Longrightarrow y = -3 - x$

If $x = 0, y = -3 - x \Longrightarrow y = -3 - 0 \Longrightarrow y = -3$; point $(0, -3)$

If $x = -3, y = -3 - x \Longrightarrow y = -3 - (-3) \Longrightarrow y = -3 + 3 \Longrightarrow y = 0$; point $(0, -3)$

| $x$ | 0 | −3 |
|---|---|---|
| y | −3 | 0 |
| point | (0,−3) | (−3 ,0) |

Plotting the points $C(0, -3)$ and D(−3,0)




We plot the points on graph. from the graph the points intersect at the point $(x, y) = (-15,12)$

$\therefore x = -15$ and $y = 12$

3.Solve the following pair of linear equation graphically $2x + y = 6; 4x - 2y = 4$

Solution: The given pair of linear equation:

$2x + y = 6$ ---------- (i) $\quad$ $4x - 2y = 4$---------- (ii)

for eq (i ): $2x + y = 6 \Rightarrow y = 6 - 2x$

If $x = 0$ , $y = 6 - 2x \Rightarrow y = 6 - 2 \times 0 \Rightarrow y = 6 - 0 \Rightarrow y = 6$; point$(0, 6)$

If $x = 3$ , $y = 6 - 2x \Rightarrow y = 6 - 2 \times 3 \Rightarrow y = 6 - 6 \Rightarrow y = 0$; point$(3, 0)$

| $x$ | 0 | 3 |
|---|---|---|
| y | 6 | 0 |
| point | (0,6) | (3, 0) |

Plotting the points A (0,6)and B(3,0)

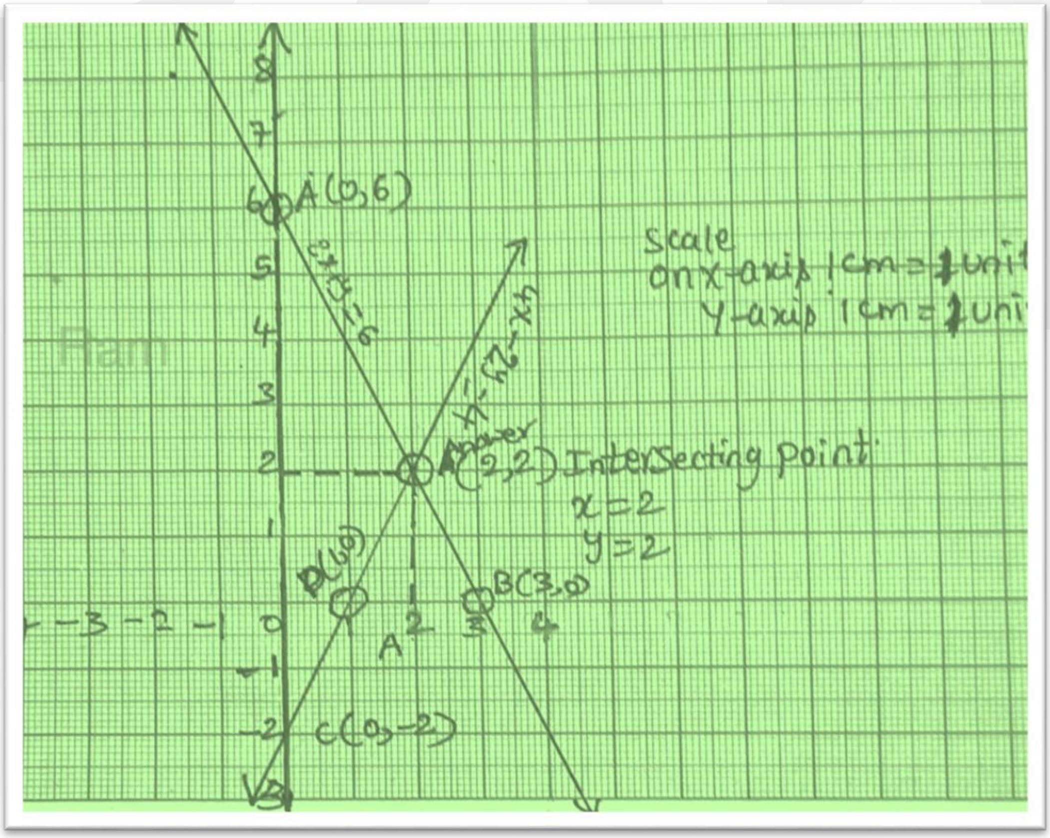
for eq (ii ): $4x - 2y = 4 \Rightarrow y = \frac{4x - 4}{2}$

If $x = 0$, $y = \frac{4x-4}{2} \Rightarrow y = \frac{4\times0-4}{2} \Rightarrow y = \frac{0-4}{2} \Rightarrow y = \frac{-4}{2} \Rightarrow y = -2$ ; point $(0, -2)$

If $x = 1$, $y = \frac{4x-4}{2} \Rightarrow y = \frac{4\times1-4}{2} \Rightarrow y = \frac{4-4}{2} \Rightarrow y = \frac{0}{2} \Rightarrow y = 0$ ; point $(1,0)$

| $x$ | 0 | 1 |
|---|---|---|
| y | −2 | 0 |
| point | (0,−2) | (1,0) |

Plotting the points C (0,−2)and D(1,0)




We plot the points on graph. from the graph the points intersect at the point $(x, y) = (2,2)$

$\therefore x = 2$ and $y = 2$

4.Form the pair of linear equations in the following situation and find their solution graphically.

5 pencils and 7 pens together cost 50 Rupees, whereas 7 pencils and 5 pens together cost 46 Rupees.

Solution: Let us assume cost of 1 pencil be $x$ and cost of 1 pen be $y$.

The cost of 5 pencils and 7 pens is ₹ 50. Mathematically, $5x + 7y = 50$

And, the cost of 7 pencils and 5 pens is ₹ 50. Mathematically, $7x + 5y = 46$

The pair of linear equations:

$5x + 7y = 50$ --------- -(i)

$7x + 5y = 46$ ----------(ii)

The algebraic representation for equation (i) is:

$5x + 7y = 50 \Rightarrow 7y = 50 - 5x \Rightarrow y = \frac{50 - 5x}{7}$

and the algebraic representation for equation (ii) is:

$7x + 5y = 46 \Rightarrow 5y = 46 - 7x \Rightarrow y = \frac{46 - 7x}{5}$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

for eq (i): $5x + 7y = 50$

If $x = 3$ $then$ $y = \frac{50 - 5x}{7} \Rightarrow y = \frac{50 - 5 \times 3}{7} \Rightarrow y = \frac{50 - 15}{7} \Rightarrow y = \frac{35}{7} \Rightarrow y = 5$; point (3,5)

If $x = -4$ $then$ $y = \frac{50 - 5x}{7} \Rightarrow y = \frac{50 - 5 \times (-4)}{7} \Rightarrow y = \frac{50 + 20}{7} \Rightarrow y = \frac{70}{7} \Rightarrow y = 10$; point (3,5)

| $x$ | 3 | $-4$ |
|---|---|---|
| $y = \frac{50 - 5x}{7}$ | 5 | 10 |
| Point | (3,5) | (−4,10) |

Plotting the points A (3,5)and (−4,10)

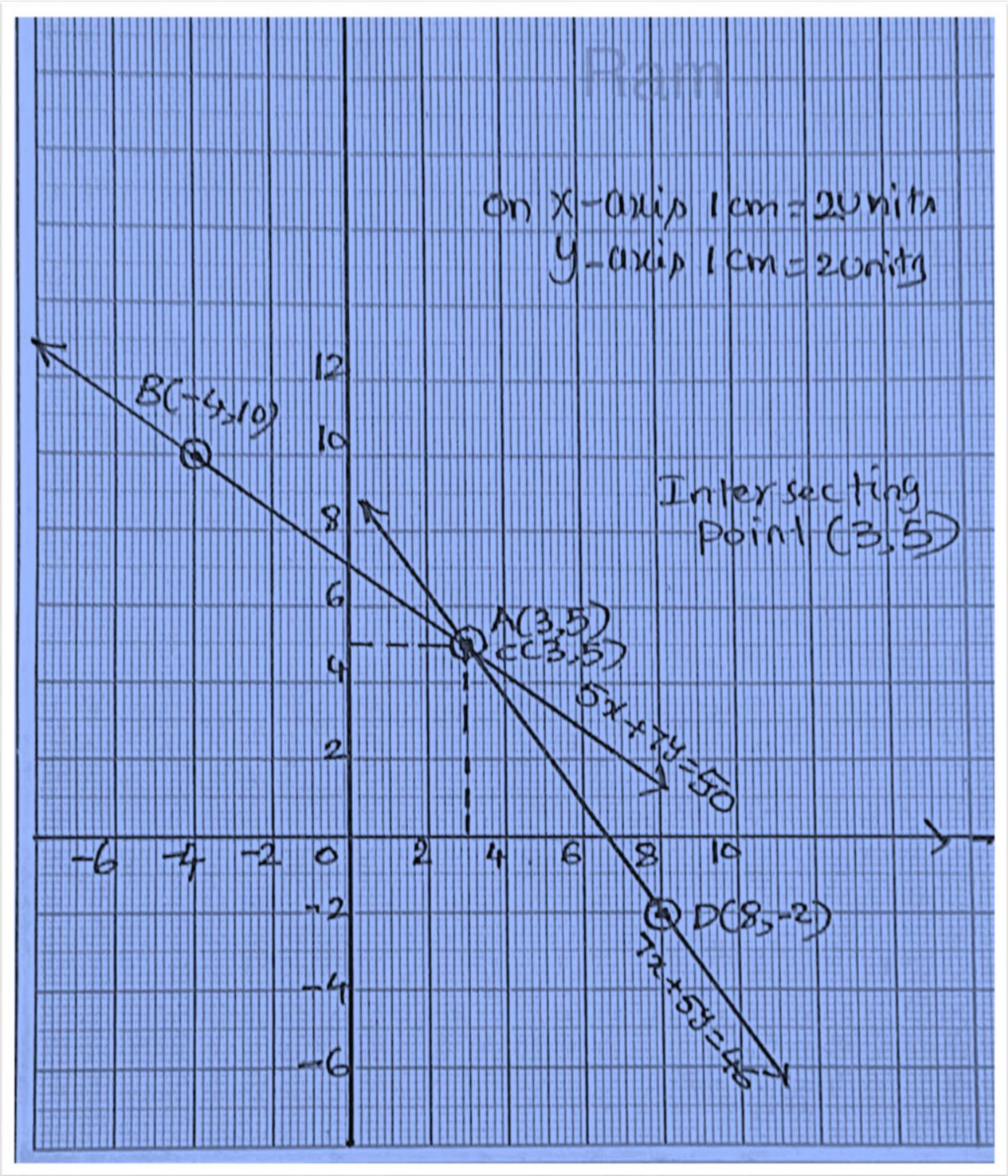
for eq (ii): $7x + 5y = 46$

If $x = 3$ $then$ $y = \frac{46 - 7x}{5} \Rightarrow y = \frac{46 - 7 \times 3}{5} \Rightarrow y = \frac{46 - 21}{5} \Rightarrow y = \frac{25}{5} \Rightarrow 5$ $y = 5$; point (3,5)

If $x = 8$ $then$ $y = \frac{46 - 7x}{5} \Rightarrow y = \frac{46 - 7 \times 8}{5} \Rightarrow y = \frac{46 - 56}{5} \Rightarrow y = \frac{-10}{5} \Rightarrow y = -2$; point (8, −2)

| $x$ | 3 | 8 |
|---|---|---|
| $y = \frac{46 - 7x}{5}$ | 5 | $-2$ |
| Point | (3,5) | (8,−2) |

Plotting the points C (3,5)and D( 8, −2)




From graph Solution: $(x, y) = (3,5)$

Cost of one pencil = ₹ 3 and Cost of one pen = ₹ 5

5.Solve the following pair of linear equation graphically $x + 3y = 6$ ; $2x - 3y = 12$

Solution: The given pair of linear equations:

$x + 3y = 6$ ---------- (i )

$2x - 3y = 12$---------- (ii)

for eq (i ): $x + 3y = 6 \Rightarrow y = \frac{6-x}{3}$

If $x = 0$ , $y = \frac{6-x}{3} \Rightarrow y = \frac{6-0}{3} \Rightarrow y = \frac{6}{3} \Rightarrow y = 2$ ; point $(0, 2)$

If $x = 6$ , $y = \frac{6-x}{3} \Rightarrow y = \frac{6-6}{3} \Rightarrow y = \frac{0}{3} \Rightarrow y = 0$ ; point $(6, 0)$

| $x$ | 0 | 6 |
|---|---|---|
| y | 2 | 0 |
| point | (0,2) | (6, 0) |

Plotting the points A (0,2)and B(6,0)

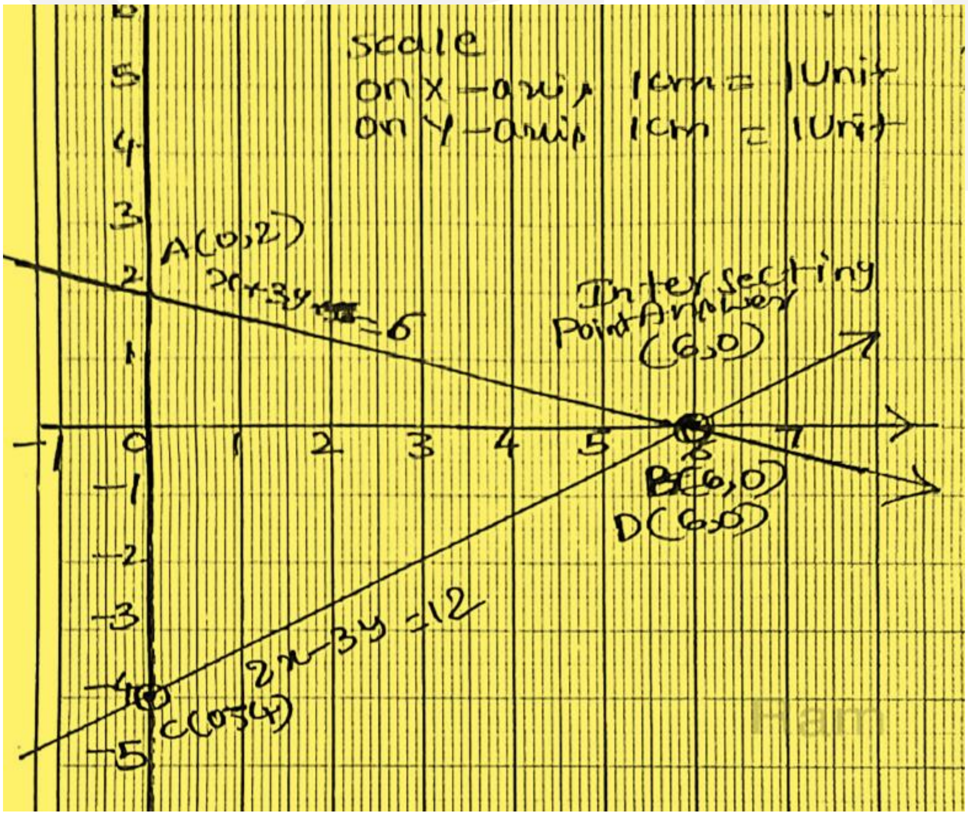
eq (ii ): $2x - 3y = 12 \Rightarrow y = \frac{2x-12}{3}$

If $x = 0, y = \frac{2x-12}{3} \Rightarrow y = \frac{2\times 0 - 12}{3} \Rightarrow y = \frac{0-12}{3} \Rightarrow y = \frac{-12}{3} \Rightarrow y = -4$ ; point $(0, -4)$

If $x = 6, y = \frac{2x-12}{3} \Rightarrow y = \frac{2\times 6 - 12}{3} \Rightarrow y = \frac{12-12}{3} \Rightarrow y = \frac{10}{3}$ $y = 0$ ; point (6,0)

| $x$ | 0 | 6 |
|---|---|---|
| y | $-4$ | 0 |
| point | (0,$-4$) | (6,0) |

Plotting the points C $(0, -4)$and D(6,0)




We plot the points on graph. from the graph the points intersect at the point $(x, y) = (6,0)$

$\therefore x = 6$ and $y = 0$

6."10 students of class X took part in a Mathematics Quiz. If the number of girls is 4 more than the more of boys". Then form the pair of linear equations in the above situation and find the number of boys and girls took part in the quiz by graphically.

Solution: Let the number of boys participated in quiz be '$x$' and number of girls participated be 'y'

Total number of boys and girls is: $x + y = 10$

Number of girls is 4 more than the number of boys, i.e. $y = x + 4$

$$x + y = 10 \text{----------- (i)}$$
$$-x + y = 4 \text{ ----------- (ii)}$$

Therefore, the algebraic representation for equation (i): $x + y = 10 \Rightarrow y = 10 - x$

the algebraic representation for equation (ii): $-x + y = 4 \Rightarrow y = x + 4$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

for eq (i ): $x + y = 10 y = 10 - x$

If $x = 2, y = 10 - x \Rightarrow y = 10 - 2 = 8$; point (2,8)

If $x = 7, y = 10 - x \Rightarrow y = 10 - 7 = 3$; point (7,3)

| $x$ | 2 | 7 |
|---|---|---|
| $y = 10 - x$ | 8 | 3 |
| Point | (2,8) | (7,3) |

Plotting the points A (2,8)and B(7,3)

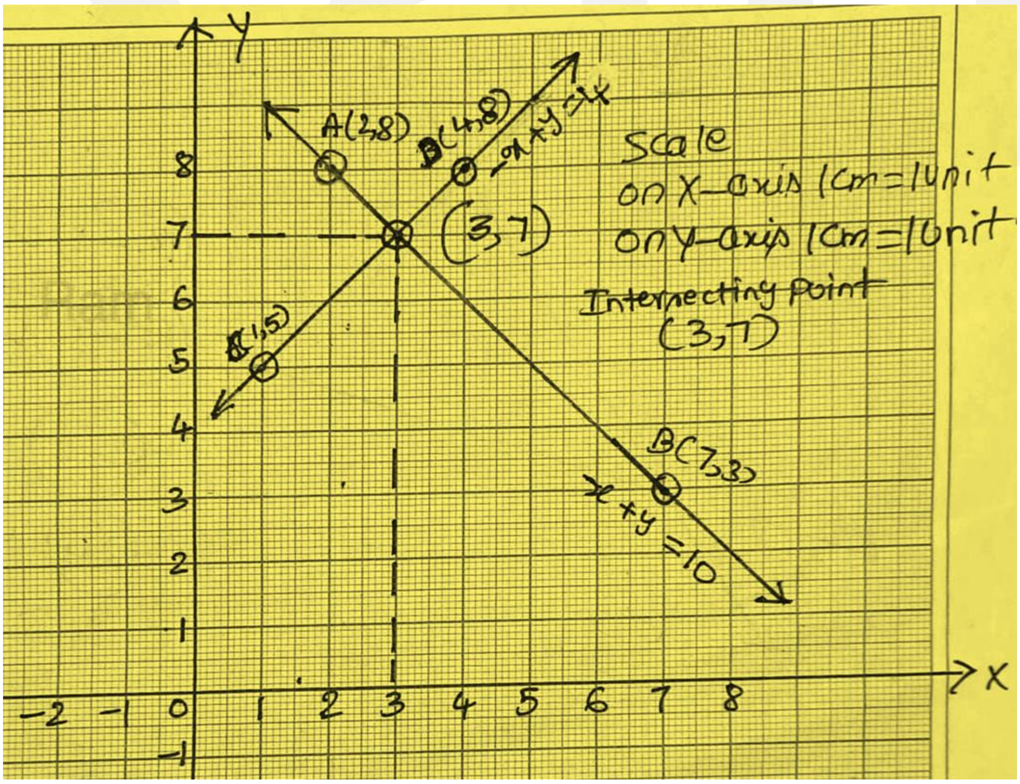
for equation (ii): $-x + y = 4 \Rightarrow y = x + 4$

If $x = 1, \Rightarrow y = x + 4 \Rightarrow y = 1 + 4 \Rightarrow y = 5$ ; point (1,5)

If $x = 4, y = x + 4 \Rightarrow y = 4 + 4 = 8 \Rightarrow y = 8$; point (4,8)

| $x$ | 1 | 4 |
|---|---|---|
| $y = x + 4$ | 5 | 8 |
| Point | (1,5) | (4,8) |

Plotting the points C(1,5)and D(4,8)




From graph solution $(x, y) = (3,7)$

Number of boys $= 3$ , Number of girls $= 7$

## LEVEL-2: SHINING STAR

### 8 MARKS QUESTION ANSWERS

1.Form the pair of linear equations for the following problem and find the solution by graphical method "sum of two numbers is 10 and their difference is 2".

Solution: Let the two numbers be '$x$' and 'y' and assume that $x > y$

Given sum of two numbers is 10, i. e, $x + y = 10$------------- (i)

and their difference is 2.i.e. $x - y = 2$ ------------- (ii)

Therefore, the algebraic representation for equation (i): $x + y = 10 \Longrightarrow y = 10 - x$

the algebraic representation for equation (ii): $x - y = 4 \Longrightarrow y = x - 4$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

for eq (i ): $x + y = 10 \Longrightarrow y = 10 - x$

If $x = 2, y = 10 - x \Longrightarrow y = 10 - 2 = 8$; point (2,8)

If $x = 7, y = 10 - x \Longrightarrow y = 10 - 7 = 3$; point (7,3)

| $x$ | 2 | 7 |
|---|---|---|
| $y = 10 - x$ | 8 | 3 |
| Point | (2,8) | (7,3) |

Plotting the points A (2,8)and B(7,3)

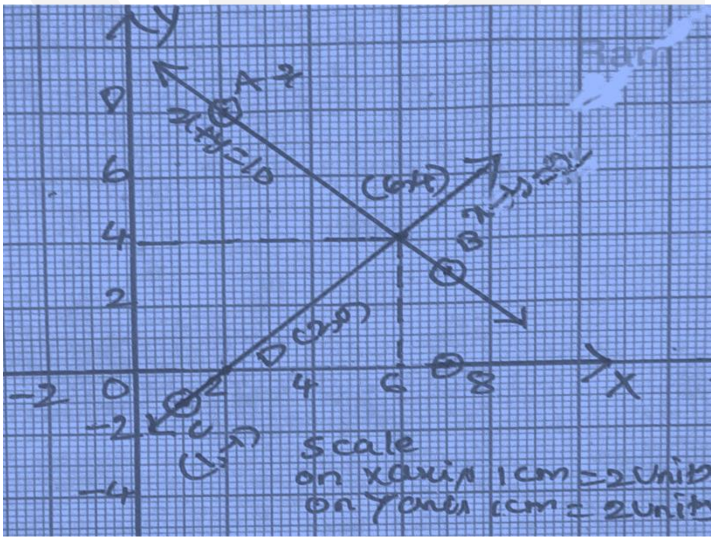
for eq (ii): $x - y = 2 \Longrightarrow y = x - 2$

If $x = 1, y = x - 2 \Longrightarrow y = 1 - 2 \Longrightarrow y = -1$; point $(1, -1)$

If $x = 2, \Longrightarrow y = 2 - 2 \Longrightarrow y = 0$; point (2,0)

| $x$ | 1 | 2 |
|---|---|---|
| $y = x - 2$ | $-1$ | 0 |
| Point | (1,–1) | (2,0) |

Plotting the points C$(1, -1)$and D(2,0)




From the graph the intersecting point of two linear equations $(x, y)$ =(6,4)

i.e., $x = 6$ and $y = 4$

The two numbers $x = 6$ and $y = 4$

2.Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden graphically.

Solution: Let us assume that the length of the garden be $x$ meters and breadth be $y$ meters

perimeter of rectangular garden = 2 (length + breadth) = 2 ( $x$ + y)

Given Half the perimeter of rectangular garden = 36

i.e., $\frac{1}{2} \times 2\,(x + y) = 36 \Rightarrow x + y = 36$ .......... eq (i)

and length is 4 m more than width i.e.,$x = y + 4 \Rightarrow x - y = 4$ .......... eq (ii)

Therefore, the algebraic representation for equation (i): $x + y = 36 \Rightarrow y = 36 - x$

the algebraic representation for equation (ii): $x - y = 4 \Rightarrow y = x - 4$

Let us represent these equations graphically. For this, we need at least two solutions for each equation.

We give these solutions in table shown below.

for eq (i ): $x + y = 36 \Rightarrow y = 36 - x$

If $x = 12, y = 36 - x \Rightarrow y = 36 - 12 = 24$; point (12,24)

If $x = 20, y = 36 - x \Rightarrow y = 36 - 20 = 16$; point 20,16)

| $x$ | 12 | 20 |
|---|---|---|
| $y = 36 - x$ | 24 | 16 |
| Point | (12,24) | (20,16) |

Plotting the points A (12,24)and B(20,16)

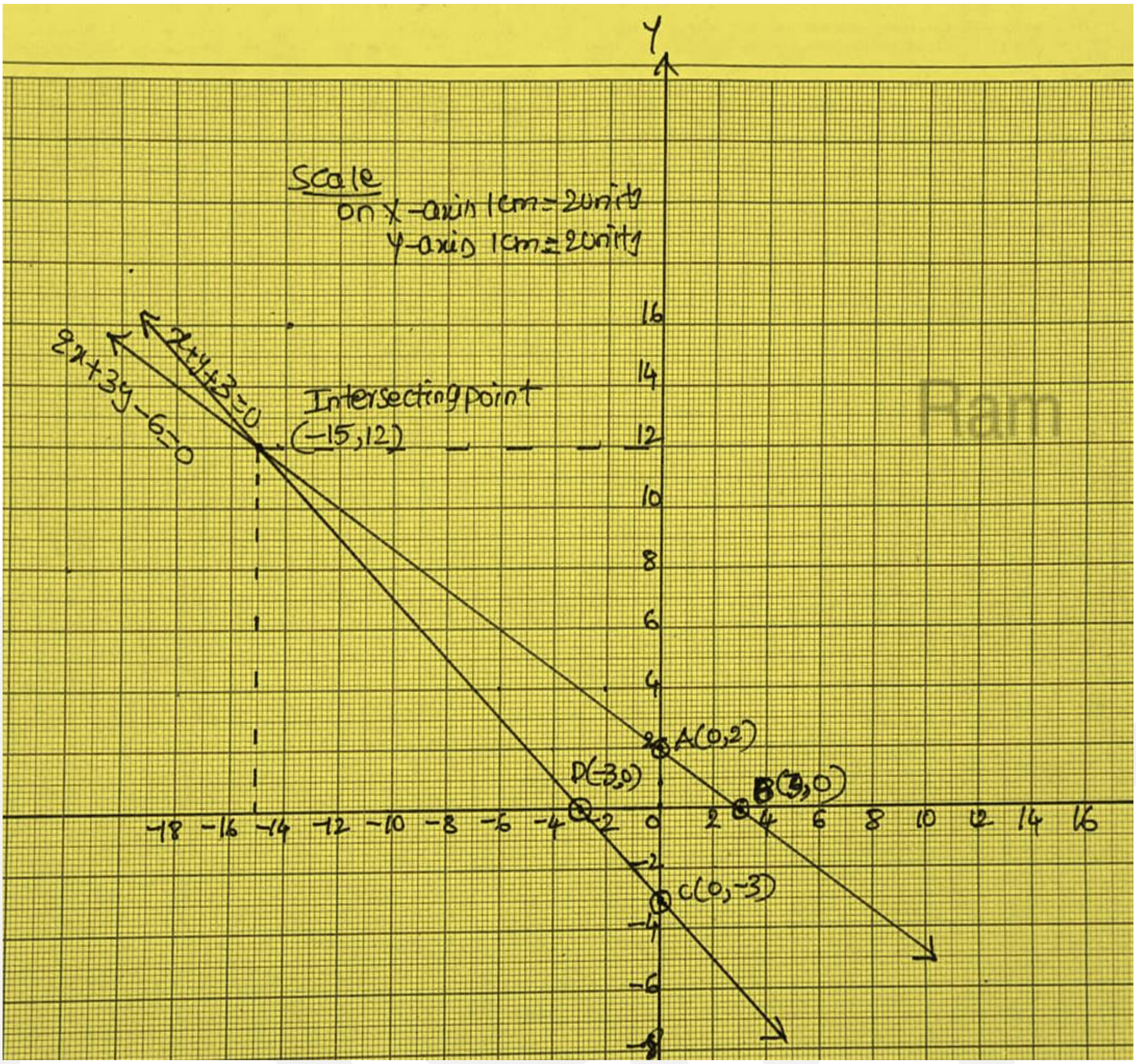
for equation (ii): $x - y = 4 \Rightarrow y = x - 4$

If $x = 0, \Rightarrow y = x - 4 \Rightarrow y = 0 - 4 \Rightarrow y = -4$ ; point $(0, -4)$

If $x = 4, \Rightarrow y = x - 4 \Rightarrow y = 4 - 4 \Rightarrow y = 0$ ; point (4,0)

| $x$ | 0 | 4 |
|---|---|---|
| $y = x - 4$ | $-4$ | 0 |
| Point | (0,−4) | (4,0) |

Plotting the points C(0, −4)and D(4,0)

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We plot the points on graph. from the graph the points intersect at the point$(x, y) = (20,16)$

∴The length of the garden $x = 20$m and the breadth $y = 16$m

3.Solve the following pair of linear equations graphically $2x + y - 5 = 0$, $3x - 2y - 4 = 0$

Solution: The given pair of linear equation:

$2x + y - 5 = 0 \implies 2x + y = 5$ ---------- (i)

$3x - 2y - 4 = 0 \implies 3x - 2y = 4$ ---------- (ii)

for eq (i ): $2x + y = 5 \implies y = 5 - 2x$

If $x = 0$ , $y = 5 - 2x \implies y = 5 - 2 \times 0 = 5 \implies y = 5 - 0 \implies y = 5$; point$(0, 5)$

If $x = 1$ , $y = 5 - 2x \implies y = 5 - 2 \times 1 = 5 \implies y = 5 - 2 \implies y = 3$; point$(1, 3)$

| $x$ | 0 | 1 |
|---|---|---|
| $y = 5 - 2x$ | 5 | 3 |
| point | (0,5) | (1,3) |

Plotting the points A (0,5)and B(1,3)

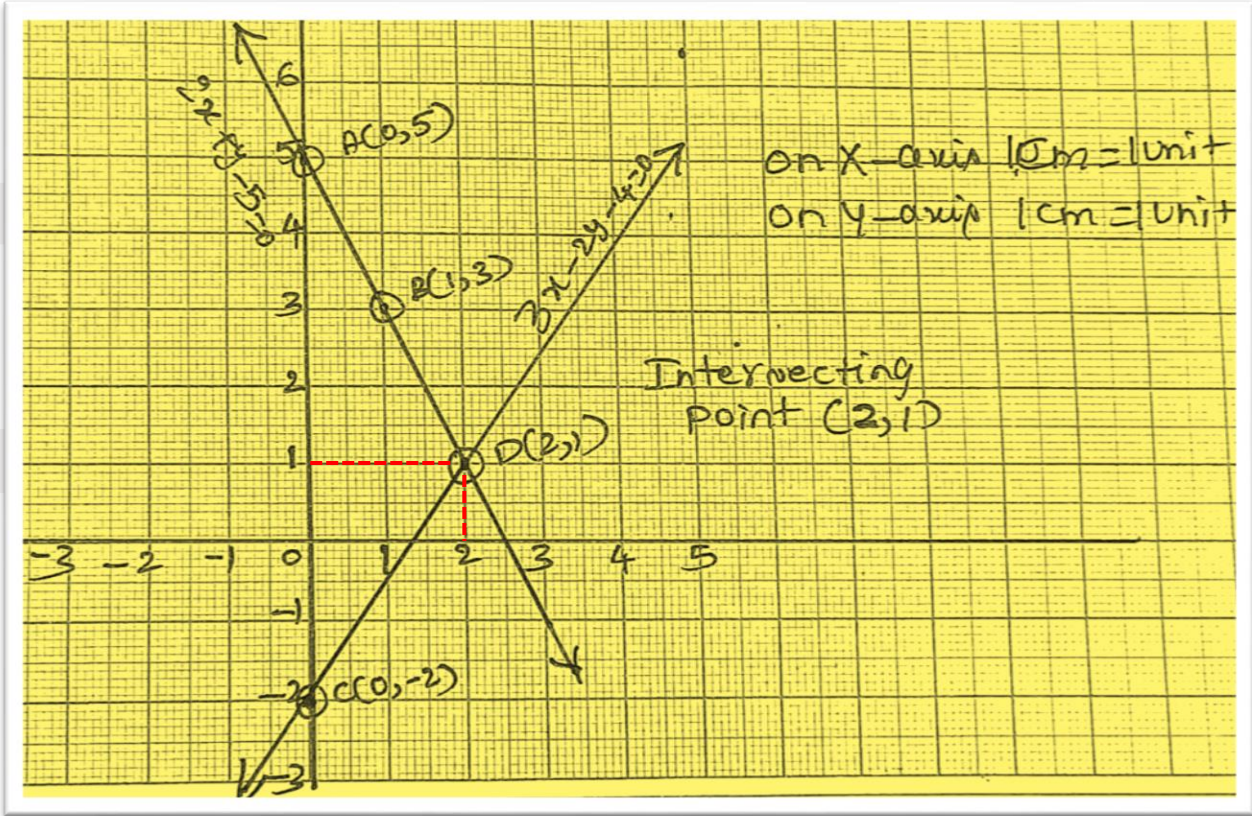
for eq (ii ): $3x - 2y = 4 \Rightarrow y = \dfrac{3x-4}{2}$

If $x = 0, \Rightarrow y = \frac{3x-4}{2} \Rightarrow y = \frac{3\times0-4}{2} \Rightarrow y = \frac{0-4}{2} \Rightarrow y = \frac{-4}{2} \Rightarrow y = -2$ ; point $(0,-2)$

If $x = 2, \Rightarrow y = \frac{3x-4}{2} \Rightarrow y = \frac{3\times2-4}{2} \Rightarrow y = \frac{6-4}{2} \Rightarrow y = \frac{2}{2} \Rightarrow y = 1$ ; point $(2,1)$

| $x$ | 0 | 2 |
|---|---|---|
| $y = \dfrac{3x-4}{2}$ | $-2$ | 1 |
| point | $(0,-2)$ | $(2,1)$ |

Plotting the points C $(0,-2)$and D$(2,1)$




We plot the points on graph. from the graph the points intersect at the point $(x, y) = (2,1)$

$\therefore x = 2$ and $y = 1$

4.Draw the graphs of the equation $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

Solution: Given pair of linear equations are $x - y + 1 = 0$ and $3x + 2y - 12 = 0$

$x - y + 1 = 0 \Rightarrow x - y = -1$ ---------- (i)

$3x + 2y - 12 = 0 \Rightarrow 3x - 2y = 12$ ---------- (ii)

The algebraic representation for equation ( i) is:

$x - y + 1 = 0 \Rightarrow x - y = -1 \Rightarrow y = x + 1$

and the algebraic representation for equation (ii) is:

$3x + 2y - 12 = 0 \Rightarrow 3x + 2y = 12 \Rightarrow 2y = 12 - 3x \Rightarrow y = \frac{12-3x}{2}$

Let us represent these equations graphically. For this, we need at least two solutions for each equation. We give these solutions in table shown below.

for eq (i): $x - y = -1$

If $x = 0$ $then$ $y = y = x + 1 \Rightarrow y = 0 + 1 \Rightarrow y = 1$; point (0,1)

If $x = 1$ $then$ $y = 1 + 1 \Rightarrow y = 2$ ; point (1,2)

| $x$ | 0 | 1 |
|---|---|---|
| $y = x + 1$ | 1 | 2 |
| point | (0,1) | (1,2) |

Plotting the points A (0,1)and D( 1,2)

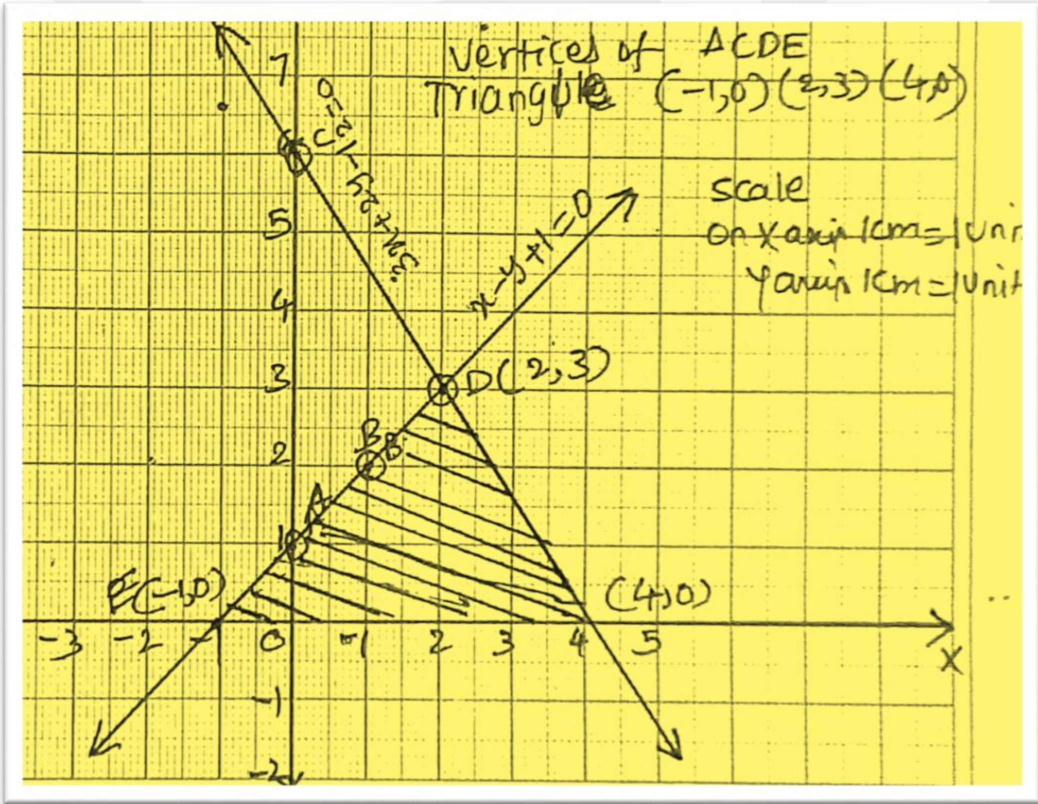
for eq (ii): $3x - 2y = 12$

If $x = 0$ $then$ $y = \frac{12-3x}{2} \Rightarrow y = \frac{12-3\times0}{2} \Rightarrow y = \frac{12-0}{2} \Rightarrow y = \frac{12}{2} \Rightarrow y = 6$; point (0,6)

If $x = 2$ $then$ $y = \frac{12-3x}{2} \Rightarrow y = \frac{12-3\times2}{2} \Rightarrow y = \frac{12-6}{2} \Rightarrow y = \frac{6}{2} \Rightarrow y = 3$; point (2,3)

| $x$ | 0 | 2 |
|---|---|---|
| $y = \frac{12-3x}{2}$ | 6 | 3 |
| point | (0,6) | (2,3) |

Plotting the points C (0,6)and D( 2,3)




From graph, vertices of the triangle are $(-1, 0)$, $(4, 0)$, and $(2, 3)$

## 1 MARK QUESTIONS ANSWERS

1.Write the general form of linear equation in two variables.

The general form of linear equation in two variables is $ax + by + c = 0$ where $a$, $b$ and c are integers

2."The cost of 2 pens and 5 pencils is Rs. 20". Express this data as a linear equation.

Answer: Let the cost of one pencil be ₹ '$x$' and cost of one pen be ₹ '$y$'

From the given condition, $2x + 5y = 50$

3.Match the following.

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are pair of linear equations.

i) Unique solution ( ) a) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

ii) No solution ( ) b) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

iii) Infinitely many solutions ( ) c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

A)1-a, 2-b, 3-c B) i-b, ii-c, iii-a C)1-c, ii-b, iii D) i-b, ii-a, iii-c

4.Write another linear equation is parallel to $3x - 2y + 4 = 0$

$3x - 2y + 5 = 0$

5. On comparing ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent or inconsistent $2x - 3y = 8$; $4x - 6y = 9$

$2x - 3y = 8 \Rightarrow 2x - 3y - 8 = 0$ ;

$4x - 6y = 9 \Rightarrow 4x - 6y - 9 = 0$

comparing with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

$2x - 3y - 8 = 0$ $here\ a_1 = 2, b_1 = -3, c_1 = -8$

and $4x - 6y - 9 = 0$ $here\ a_2 = 4, b_2 = -6, c_2 = -9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$ $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9},$ We clearly observe that $\frac{1}{2} = \frac{1}{2} \neq \frac{8}{9}$

i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Hence the given pair of linear equations are parallel and inconsistent

6.Find the value of k for which the following equations have infinitely many solutions

$4x + 5y = 3$ $kx + 15y = 9$

Solution:

Given pair of linear equations: $4x + 5y = 3 \Rightarrow 4x + 5y - 3 = 0$

$Kx + 15y = 9 \Rightarrow kx + 15y - 9 = 0$

comparing with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

$4x + 5y - 3 = 0$ $here\ a_1 = 4, b_1 = 5, c_1 = -3$

and $kx + 15y - 9 = 0$ $here\ a_2 = k, b_2 = 15, c_2 = -9$

$\frac{a_1}{a_2} = \frac{4}{k}, \frac{b_1}{b_2} = \frac{5}{15} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-9} = \frac{1}{3}$

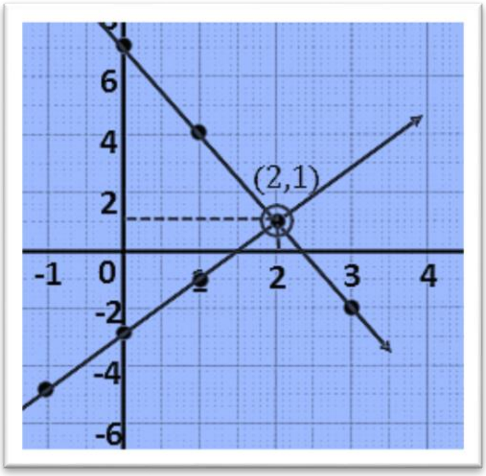
Given equations have infinitely many solutions hence $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{4}{k} = \frac{1}{3} = \frac{1}{3}$ we take $\frac{4}{k} = \frac{1}{3} \Rightarrow k = 3 \times 4 = 12$

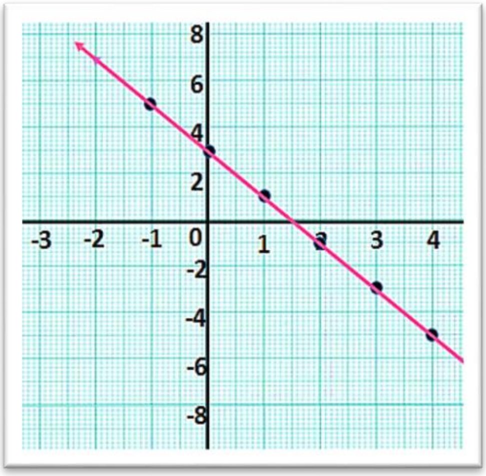
$\therefore k = 12$

7.Draw the rough diagrams of pair of linear equations in two variables which have unique solution and infinitely many solutions.

unique solution: $3x + y = 7,\ 2x - y = 3$




infinitely many solutions: $2x + y = 3 : 6x + 3y = 9$



8.Form the pair of linear equations for the following data "The larger of two supplementary angles exceeds the smaller by 18°.

Supplementary angles are two angles with a sum of 180 and assuming the angles as $x$ and $y$ , two linear equations can be formed for the known situation.

Let the larger angle $= x$ and smaller angle $= y$ Since the angles are supplementary $x + y = 180°$

Larger angle exceeds the smaller by 18° $x = y + 18° \implies x - y = 18°$

**9.Assertion (A):** The pair of equations $2x - 3y = 1$ and $3x + y = 7$ are intersecting lines.

**Reason (R):** The pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are intersecting if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Now, choose the correct answer.

A) Both A and R are true ✓

B) Both A and R are false.

C) A is true, but R is false

D) A is false, but R is true

10. **Assertion 1** ($A_1$)**:** Pair of linear equations have infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

**Assertion2** ($A_2$)**:** Pair of linear equations have no solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Now, choose the correct answer.

A) Both $A_1$ and $A_2$ are true ✓

B) Both $A_1$ and $A_2$ are false.

C) $A_1$ is true, $A_2$ is false

D) $A_1$ is false, but $A_2$ is true

# 4.QUADRATIC EQUATIONS

[7 Marks] [1 + 2 + 4 = 7 M]

## LEVEL-1: RISING STAR

### 4 MARKS QUESTIONS ANSWERS

1.State the conditions under which a quadratic equation will have

i) Two real and distinct roots  ii) Two equal roots

Solution: i) Two real and distinct roots

A quadratic equation $ax^2 + bx + c = 0$, will have two real and distinct roots

if the discriminant (D): $b^2 - 4ac > 0$

ii) Two equal roots

A quadratic equation $ax^2 + bx + c = 0$, will have two equal roots

if the discriminant (D): $b^2 - 4ac = 0$

2. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$, and hence find the nature of roots.

Find them if they are real

Solution:

The given equation $3x^2 - 2x + \frac{1}{3} = 0$, is of the form $ax^2 + bx + c = 0$,

$Here\ a = 3,\ b = -2\ and\ c = \frac{1}{3}$

The discriminant (D): $b^2 - 4ac = (-2)^2 - 4 \times 3 \times \frac{1}{3}$

$= 4 - 4 = 0$

discriminant (D): $b^2 - 4ac = 0$ So, the given quadratic equation has two real and equal roots

3.State the nature of roots of the following quadratic equations.

$i) 2x^2 - 3x + 5 = 0$  $ii) 2x^2 - 6x + 3 = 0$

Solution:

$i) 2x^2 - 3x + 5 = 0$

The given equation $2x^2 - 3x + 5 = 0$, is of the form $ax^2 + bx + c = 0$,

$Here\ a = 2,\ b = -3\ and\ c = 5$

Therefore, the discriminant $b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5$

$= 9 - 40 = -31 < 0$

discriminant (D): $b^2 - 4ac < 0$ So, the given quadratic equation has no real roots

$ii) 2x^2 - 6x + 3 = 0$

The given equation $2x^2 - 6x + 3 = 0$ is of the form $ax^2 + bx + c = 0$,

$Here\ a = 2,\ b = -6\ and\ c = 3$

the discriminant $b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

The discriminant $b^2 - 4ac > 0$ So, the given quadratic equation has two real and distinct roots

4.Express the roots of the quadratic equation $3x^2 - 4\sqrt{3}\,x + 4 = 0$ by quadratic formula method.

Solution: The given equation $3x^2 - 4\sqrt{3}\,x + 4 = 0$, is of the form $ax^2 + bx + c = 0$,

Here $a = 3,\ b = -4\sqrt{3}\ and\ c = 4$

By using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-4\sqrt{3}) \pm \sqrt{(-4\sqrt{3})^2 - 4\times3\times4}}{2\times3}$$

$$= \frac{4\sqrt{3} \pm \sqrt{48-48}}{6} = \frac{4\sqrt{3} \pm 0}{6}$$

$$\Rightarrow x = \frac{4\sqrt{3} \pm 0}{6} \Rightarrow x = \frac{4\sqrt{3}+0}{6} \text{ and } x = \frac{4\sqrt{3}-0}{6}$$

$$\Rightarrow x = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} \text{ and } x = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}$$

∴ The roots of the given quadratic equation $x = \frac{2\sqrt{3}}{3}$ *and* $\frac{2\sqrt{3}}{3}$

5.Find the nature of roots of quadratic equation $2x^2 + x - 6 = 0$. If the real roots exist find them.

Solution: The given equation $2x^2 + x - 6 = 0$ is of the form $ax^2 + bx + c = 0$,

$Here\ a = 2,\ b = 1\ and\ c = -6$

The discriminant $b^2 - 4ac = (1)^2 - 4 \times 2 \times (-6)$

$= 1 + 48 = 49 > 0$

discriminant $(D)$: $b^2 - 4ac > 0$ so it has two real and distinct roots

$2x^2 + x - 6 = 0 \Longrightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0 \Longrightarrow (x+2)(2x-3) = 0$

therefore, $(x+2) = 0$ or $(2x-3) = 0$

$i.e., x + 2 = 0 \Rightarrow x = -2$ or $2x - 3 = 0 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

∴ The roots of $2x^2 + x - 6 = 0$ *are* $-2$ and $\frac{3}{2}$

6.Find the value of ‘ K’ for the quadratic equation Kx (x – 2) + 6 = 0 if it has two equal roots.

Solution: Given quadratic equation: $kx(x-2) + 6 = 0$

$kx^2 - 2kx + 6 = 0$ is of the form $ax^2 + bx + c = 0$,

$Here\ a = k, b = -2k\ and\ c = 6.$

Given it has two equal roots. So the discriminant$(D)$: $b^2 - 4ac = 0$

$(-2k)^2 - 4 \times k \times 6 = 0 \Rightarrow 4k^2 - 24k = 0$

$\Rightarrow 4k(k-6) = 0 \Rightarrow 4k = 0\ or\ k = 6\ i.e.,\ k = 0\ or\ k = 6$

If we consider the value of k $=$ 0, then the equation will not longer be quadratic.

$\therefore k = 6$

## 2 MARKS QUESTION ANSWERS

1.Develop the following situation in the form of quadratic equation.

"The product of two consecutive positive integers is 306".

Solution: Let the first positive integer $= x$

then the next consecutive positive integer $=$ x + 1

Given : product of two consecutive positive integers $=$ 306

$x(x+1) = 306 \Rightarrow x^2 + x = 306 \Rightarrow x^2 + x - 306 = 0$

∴ The given situation is in the form of quadratic equation: $x^2 + x - 306 = 0$

2.Analyze whether $(x-2)^2+1=2x-3$ is a quadratic equation or not?

Solution: $given\ (x-2)^2+1=2x-3$

$\Rightarrow x^2-4x+4+1=2x-3$

$\Rightarrow x^2-4x+5=2x-3$

$\Rightarrow x^2-4x-2x+5+3=0 \Rightarrow x^2-6x+8=0$

$x^2-6x+8=0$ It is of the form $a\,x^2+bx+c=0$.

$\therefore$ *The* given equation is a quadratic equation .

3.Detect the nature of roots of $2x^2+3x+5=0$ by calculating discriminant.

Solution: The given equation $2x^2+3x+5=0$, is of the form $ax^2+bx+c=0$,

Here $a$ = 2, $b$= 3 and c = 5.

The discriminant ( D): $b^2-4ac=(3)^2-4\times2\times5=9-40=-31<0$

discriminant : $D=-31<0$ so it has no real roots

4.Find the value of 'K' for quadratic equation $2x^2+Kx+3=0$ if it has two equal real roots.

Solution: Given quadratic equation: $2x^2+kx+3=0$

It is of the form $ax^2+bx+c=0$, Here $a=2, b=k$ and $c=3$.

Given that it has two equal roots. So the discriminant( $D$): $b^2-4ac=0$

$(k)^2-4\times2\times3=0 \Rightarrow k^2-24=0 \Rightarrow k=\pm\sqrt{24}=\pm\sqrt{4\times6}=\pm2\sqrt{6}$

$\therefore k=\pm2\sqrt{6}$

5.Find the nature of roots of quadratic equation $3x^2-4\sqrt{3}\,x+4=0$

Solution: The given equation $3x^2-4\sqrt{3}\,x+4=0$, is of the form $ax^2+bx+c=0$,

$Here\ a=3,\ b=-4\sqrt{3}\ and\ c=4$

The discriminant (D): $b^2-4ac=\left(-4\sqrt{3}\right)^2-4\times3\times4=48-48=0$

discriminant : $D=0$ *so it has two equal real roots*

6.Analyse whether $x^2+3x+1=(x-2)^2$ is a quadratic equation or not?

Solution: $x^2+3x+1=(x-2)^2$

$\Rightarrow x^2+3x+1=x^2-4x+4$

$\Rightarrow x^2+3x+1=x^2-4x+4$

$\Rightarrow 3x+1=-4x+4$

$\Rightarrow 3x+4x=4-1 \Rightarrow 7x=3 \Rightarrow 7x-3=0$ It's in linear form

Hence the given equation is not a quadratic equation

7.Detect the roots of quadratic equation $6x^2-x-2=0$

Solution: The given equation $6x^2-x-2=0$, is of the form $ax^2+bx+c=0$,

Here $a=6,\ b=-1\ and\ c=-2$

the discriminant $b^2-4ac=(-1)^2-4\times6\times(-2)=1+48=49>0$

discriminant ($D$): $b^2-4ac>0$ so it has two real and distinct roots

By using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-(-1)\pm\sqrt{49}}{2\times6}=\frac{1\pm7}{12}\Rightarrow x=\frac{1\pm7}{12}\Rightarrow x=\frac{1+7}{12}=\frac{8}{12}=\frac{2}{3}\text{ and } x=\frac{1-7}{12}=\frac{-6}{12}=\frac{-1}{2}$$

$\therefore$ The roots of $6x^2-x-2=0$ are $\frac{2}{3}$ and $\frac{-1}{2}$

## LEVEL-2: SHINING STAR

### 4 MARKS QUESTION ANSWERS

1.Find the numbers whose sum is 27 and product is 182.

Solution: Let the first number be $x$, then the second number be $(27 - x)$

Given the product of two numbers $= 182$

$x(27 - x) = 182 \Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0 \Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0 \Rightarrow (x - 13)(x - 14) = 0$

$\Rightarrow (x - 13) = 0 \Rightarrow x = 13$ or $(x - 14) = 0 \Rightarrow x = 14$

If $x = 13$ then the second number $= 27 - x = 27 - 13 = 14$

or

If $x = 14$ then the second number $= 27 - x = 27 - 14 = 13$

$\therefore$ The numbers are 13 and 14

2. Find two consecutive positive integers, whose sum of squares is 365.

Solution: Let the two consecutive positive integersbe $x$ and $(x + 1)$

Given: sum of squares of two consecutive positive integers = 365

$x^2 + (x + 1)^2 = 365 \Rightarrow x^2 + (x^2 + 2x + 1) = 365$ $(\because (a + b)^2 = a^2 + 2ab + b^2)$

$\Rightarrow 2x^2 + 2x + 1 - 365 = 0 \Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow 2(x^2 + x - 182) = 0 \Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 13x - 14x - 182 = 0 \Rightarrow x(x + 13) - 14(x + 13) = 0$

$\Rightarrow (x + 13)(x - 14) = 0$

$\Rightarrow (x + 13) = 0$ or $(x - 14) = 0$

$(x + 13 = 0 \Rightarrow x = -13$ or $(x - 14) = 0 \Rightarrow x = 14$

Value of $x$ cannot be negative (because it is given that the integers are positive).

$\therefore x = 13$

The two consecutive positive integers are 13 and 14.

3.Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is 800 $m^2$. If so, find its length and breadth.

Solution: Let the breadth of rectangle $= \boldsymbol{x}$ m.

From the sum mango groove length is twice its breadth. So, length $= \boldsymbol{2x}$ m.

Area of a rectangle = length× breadth

Given Area =800$m^2$

$\boldsymbol{x \times 2x = 800 \Rightarrow 2x^2 = 800 \Rightarrow x^2 = 400 = 0 \Rightarrow \Rightarrow x^2 - 400 = 0 \Rightarrow}$

It is of the form $ax^2 + bx + c = 0$,

$Here\ a = 1, b = 0$ and $c = -400$.

the discriminant($D$): $b^2 - 4ac = (0)^2 - 4 \times 1 \times (-400) = 0 + 1600 = 1600 > 0$

discriminant($D$) $> 0$ so it has two distinct real roots

Hence it is possible to design a rectangular mango grove to the given situation.

$\boldsymbol{x^2 - 400 = 0 \Rightarrow x^2 = 400 \Rightarrow x = \pm\sqrt{400} \Rightarrow x = \pm\sqrt{400}\ x = \pm 20}$

Value of $\boldsymbol{x}$can't be negative value as it represents the breadth of the rectangle.

So $\boldsymbol{x = 20m}$ $\therefore$ breadth $= \boldsymbol{20}$ m. and length $= \boldsymbol{2 \times 20m = 40m}$

## 1 MARK QUESTION ANSWERS

1. Write the standard form of a quadratic equation.

The standard form of a quadratic equation is $ax^2 + bx + c = 0$ .

where $a, b, c$ are real numbers and , $a \neq 0$, ,

2.Form a quadratic equation whose roots are real and equal.

If α and β are the real roots then the quadratic equation : $K[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$
Let $\alpha = \beta = 1$ then the quadratic equation : $k[x^2 - (1+1)x + 1 \times 1] = 0 \Rightarrow k[x^2 - 2x + 1] = 0$

if $k = 1$ then $k[x^2 - 2x + 1] = 0 \Rightarrow 1[x^2 - 2x + 1] = 0 \Rightarrow x^2 - 2x + 1 = 0$

then $x^2 - 2x + 1 = 0$ is the quadratic equation whose roots are real and equal

3. Form a quadratic equation which has 2 as one of its roots.

If α and β are the real roots then the quadratic equation : $K[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$
given $\alpha = 2$ and we take $\beta = 1$

hence the quadratic equation : $k[x^2 - (2+1)x + 2 \times 1] = 0 \Rightarrow k[x^2 - 3x + 2] = 0$

If $k = 1$ *then* $k[x^2 - 3x + 2] = 0 \Rightarrow 1[x^2 - 3x + 2] = 0 \Rightarrow x^2 - 3x + 2 = 0$

$\therefore x^2 - 3x + 2$ is the required quadratic equation

4.Create the following situation as quadratic equation" – The product of two consecutive Integers is 15".

Let the two consecutive positive integers *be* $x$ *and* $(x+1)$

The product of two consecutive Integers = 15
$x(x+1) = 15 \Rightarrow x^2 + x = 15 \Rightarrow x^2 + x - 15 = 0$ It is of the form $ax^2 + bx + c = 0$
$\therefore x^2 + x - 15 = 0$ *is a* quadratic equation to the given situation.

5. Generate the following situation as quadratic equation "The sum of number and its reciprocal is $\frac{1}{2}$ "
Let the first number = $x$ and its reciprocal = $\frac{1}{x}$

Given situation: The sum of number and its reciprocal = $\frac{1}{2}$

$x + \frac{1}{x} = \frac{1}{2} \Rightarrow \frac{x^2+1}{x} = \frac{1}{2} \Rightarrow 2(x^2+1) = x \Rightarrow 2x^2 + 2 = x \Rightarrow 2x^2 - x + 2 = 0$
It is of the form $ax^2 + bx + c = 0$,
$\therefore 2x^2 - x + 2 = 0$ is a quadratic equation

6. Form a quadratic equation whose roots are reciprocal to each other.

If α and β are the real roots then the quadratic equation : $K[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$
Let $\alpha = 3$ *and we take* $\beta = \frac{1}{3}$

$\alpha + \beta = 3 + \frac{1}{3} = \frac{10}{3}$ *and* $\alpha\beta = 3 \times \frac{1}{3} = 1$

hence the quadratic equation : $K[x^2 - \frac{10}{3}x + 1] = 0$

If $k = 3$*then* we get $3\left[x^2 - \frac{10}{3}x + 1\right] = 0 \Rightarrow 3x^2 - 3 \times \frac{10}{3}x + 3 \times 1 = 0 \Rightarrow 3x^2 - 3x + 3] = 0$
$3x^2 - 10x + 3 = 0$ is the required quadratic equation

7. Create a quadratic equation which has 1 as one of its roots.
Solution: If α and β are the real roots then the quadratic equation : $K[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$
Given $\alpha = 1$ and we take $\beta = 2$ *then* $\alpha + \beta = 1 + 2 = 3$ *and* $\alpha\beta = 1 \times 2 = 2$

hence the quadratic equation: $K[x^2 - 3x + 2] = 0$

*If* $k = 1$ *then* $k[x^2 - 3x + 2] = 0 \Rightarrow 1[x^2 - 3x + 2] = 0 \Rightarrow x^2 - 3x + 2 = 0$

$\therefore x^2 - 3x + 2$ is the required quadratic equation

8. Create a quadratic equation which has 2 and 3 as roots.

If α and β are the real roots then the quadratic equation : $K[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$
given $\alpha = 2$ and we take $\beta = 3$

$\alpha + \beta = 2 + 3 = 5 \ and \ \alpha\beta = 2 \times 3 = 6$

hence the quadratic equation: $k[x^2 - 5x + 6] = 0$

$if \ k = 1 \ then \ k[x^2 - 5x + 6] \Rightarrow 1[x^2 - 5x + 6] = 0 \Rightarrow x^2 - 5x + 6 = 0$

$x^2 - 5x + 6 = 0$ is the required quadratic equation

9.Generate a quadratic equation which has equal roots.

If α and β are the real roots then the quadratic equation : $K[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$
Let $\alpha = \beta = 1 \ then \ \alpha + \beta = 1 + 1 = 2 \ and \ \alpha\beta = 1 \times 1 = 1$

hence the quadratic equation : $k[x^2 - 2x + 1] = 0$

if $k = 1 \ then \ k[x^2 - 2x + 1] \Rightarrow 1[x^2 - 5x + 6] = 0 \Rightarrow x^2 - 2x + 1 = 0$

$x^2 - 2x + 1 = 0$ is quadratic equation whose roots are real and equal

10.Write the quadratic formula for finding roots of quadratic equation.

The quadratic formula for finding roots of quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

# 5. ARITHMATIC PROGRESSIONS

**[13 Marks] [1 + 4 + 8 = 13 M]**

**LEVEL-1 : RISING STAR**

## 8 MARKS QUESTION ANSWERS

1.If the sum of the first 'n' terms of an AP is $4n - n^2$. What is the first term, what is the sum of first two terms? What is the second term? Similarly find $3^{rd}$, $10^{th}$ and the $n^{th}$ terms.

Solution:

Given. Sum of first terms. $S_n = 4n - n^2$

Sum of first term , $a = S = 4 \times 1 - 1^2 = 4 - 1 = 3$

Sum of first two terms, $S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

Sum of first three terms. $S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

third term, $a_3 = S_3 - S_2 = 3 - 4 = -1$

tenth term,$a_{10} = S_{10} - S_9$

$= (4 \times 10 - 10^2) - (4 \times 9 - 9^2)$

$= (40 - 100) - (36 - 81)$

$= -60 + 45 = -15$

$\therefore$tenth term, $a_{10} = -15$

$n^{th}$term $a_n = S_n - S_{n-1}$

$= 4n - n^2 - [4(n-1) - (n-1)^2]$

$= 4n - n^2 - [4n - 4 - (n^2 - 2n + 1)]$

$= 4n - n^2 - [4n - 4 - n^2 + 2n - 1]$

$= 4n - n^2 - [6n - 5 - n^2]$

$= 4n - \cancel{n^2} - 6n + 5 + \cancel{n^2}$

$= 5 - 2n$

the $3^{rd}$, the $10^{th}$ and the $n^{th}$ terms are $-1$ , $-15$, and $5-2n$ respectively.

Alternate method:

Solution: Given Sum of first terms. $S_n = 4n - n^2$

Sum of first term , $a = S = 4 \times 1 - 1^2 = 4 - 1 = 3$

Sum of first two terms, $S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

Sum of first three terms. $S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

here a= 3, common difference $d = a_2 - a_1 = 1 - 3 = -2$

now $n^{th}$ term $a_n = 3 + (n-1) \times (-2)$ ( $\because$ n th term of an AP $a_n = a + (n-1)d$)

$a_n = 3 - 2n + 2$

$a_n = 5 - 2n$

Tenth term $a_{10} = 5 - 2 \times 10 = 5 - 20 = -15$

the $3^{rd}$, the $10^{th}$ and the $n^{th}$ terms are $-1$, $-15$, and $5 - 2n$ respectively.

2.The sum of the $4^{th}$ and $8^{th}$ terms of an AP is 24 and the sum of the $6^{th}$ and $10^{th}$ terms is 44. Find the first three terms of the AP.

Solution: Let a be the first term and d the common difference.

Given : sum of the $4^{th}$ and $8^{th}$ terms of an AP = 24

i.e, $a_4 + a_8 = 24 \Rightarrow (a + 3d) + (a + 7d) = 24 \Rightarrow 2a + 10d = 24 \Rightarrow 2(a + 5d) = 24$

$\Rightarrow a + 5d = 12 - - - eq\ (i)$

And also sum of the $6^{th}$ and $10^{th}$ terms =44

i.e, $a_6 + a_{10} = 44 \Rightarrow (a + 5d) + (a + 9d) = 44 \Rightarrow 2a + 14d = 44 \Rightarrow 2(a + 6d) = 44$

$\Rightarrow a + 7d = 22 - - - eq\ (ii)$

On subtracting eq( i) from eq (ii)

$(a + 7d) - (a + 5d) = 22 - 12 \Longrightarrow a + 7d - a - 5d = 10$

$\Rightarrow 2d = 10 \Longrightarrow d = 5$

Now substitute $d = 5$ in Eq ( i ) , we get

$a + 5d = 12 \Longrightarrow a + 5 \times 5 = 12 \Longrightarrow a + 25 = 12 \Rightarrow a = 25 - 12 \Longrightarrow a = -13$

The first three terms of an AP are $a,\ a + d\ ,\ a + 2d$

substituting $a$ and $d$ values , we get $-13\ ,\ -13 + 5$ *and* $-13 + 2 \times 5$ *i.e.* $13, -8$ *and* $-3$

$\therefore$ The first three terms of the AP are $-13, -8$ *and* $-3$

3.Subba Rao started work in 1995 at an annual salary of ₹5000 and received an increment of ₹200 each year. In which year did his income reach ₹7000 ?

Solution: From the given data , incomes received by Subbarao in the years 1995,1996,1997,...are 5000,5200,5400,...7000 Which are in AP

Here $a = 5000$ , $d = 200$

Let after $n^{th}$ year, his salary = ₹7000 hence $a_n = 7000$

$n^{th}$ *term of an AP*: $a_n = a + (n - 1)$

$a_n = 7000 \Longrightarrow 5000 + (n - 1)200 = 7000 \Longrightarrow (n - 1)200 = 7000 - 5000$

$\Longrightarrow (n - 1)200 = 2000 \Longrightarrow n - 1 = 10 \Longrightarrow n = 10 + 1 \Longrightarrow n = 11$

Therefore in $11^{th}$ year, his salary reach ₹7000 this means after 10 years of 1995

ie, 1995 + 10= 2025

$\therefore$In Year 2025 his income reached ₹7000

4.If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first 'n' terms.

Solution: Given: Sum of first 7 terms, $s_7 = 49$ *and* Sum of first 17 terms, $s_{17} = 289$

We know that sum of n terms of AP is *sum of n terms of AP* $S_n = \frac{n}{2}[2a + (n - 1)d]$

Hence $S_7 = 49 \Longrightarrow \frac{7}{2}[2a + (7 - 1)d] = 49 \Longrightarrow \frac{7}{2}[2a + 6d] = 49$

$\Rightarrow \frac{7}{2}[2(a + 3d)] = 49 \Longrightarrow 7(a + 3d) = 49 \Longrightarrow a + 3d = 7 - - - eq\ (i)$

And $S_{17} = 289 \Longrightarrow \frac{17}{2}[2a + (17 - 1)d] = 289 \Longrightarrow \frac{17}{2}[2a + 16d] = 289$

$\Rightarrow \frac{17}{2}[2(a + 8d)] = 289 \Longrightarrow 17(a + 8d) = 289 \Longrightarrow a + 8d = 17 - - - eq\ (ii)$

On subtracting eq( i) from eq (ii)

$(a + 8d) - (a + 3d) = 17 - 7 \Longrightarrow a + 8d - a - 3d = 10$

$\Longrightarrow 5d = 10 \Longrightarrow d = 2$

Now substitute $d$ =2 in Eq ( i ) , we get

$a + 3d = 7 \Longrightarrow a + 3 \times 2 = 7 \Longrightarrow a + 6 = 7 \Rightarrow a = 7 - 6 \Longrightarrow a = 1$

*sum of first n terms of AP* $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Longrightarrow S_n = \frac{n}{2}[2 \times 1 + (n - 1)2] \Longrightarrow S_n = \frac{n}{2}[2 + 2n - 2]$

$\Longrightarrow S_n = \frac{n}{2} \times 2n \Longrightarrow S_n = n \times n \Longrightarrow S_n = n^2$

∴ The sum of first n terms $S_n = n^2$

5.A sum of ₹700 is to be used to give seven cash prizes to students of a school for their over all academic performance. If each prize is ₹20 less than its preceding prize, find the value of the each of the prizes.

Solution: Let the cost of 1st prize be ₹$x$. Then the cost of 2nd prize $=$ ₹$(x - 20)$

And the cost of 3rd prize $=$ ₹$(x - 40)$

Prizes are $x, (x - 20), (x - 4) \ldots \ldots \ldots$

By observation that the costs of these prizes are in an A. P., having common difference as $-20$

and first term $a = x$ , common difference $d = -20$

Given that, $S_n = 700$

Sum of n terms of $AP$ $S_n = \frac{n}{2}[2a + (n-1)d] \Longrightarrow S_7 = \frac{7}{2}[2x + (7-1)(-20)]$

$\Longrightarrow 700 = \frac{7}{2}[2x + 6(-20)] \Longrightarrow 700 = \frac{7}{2} \times [2x - 120] \Longrightarrow 700 = \frac{7}{2} \times 2(x - 60)$

$\Longrightarrow 700 = 7(x - 60) \Longrightarrow 100 = x - 60 \Longrightarrow x = 100 + 60 = 160$

Therefore, the value of each of the prizes was ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, *and* ₹40.

## 4 MARK QUESTION ANSWERS

1.In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flowerbed?

Solution : The number of rose plants in the 1st, 2nd, 3rd, . . . , rows are : 23, 21, 19, . . . , 5

common difference $d = a_2 - a_1 = 21 - 23 = -2$ *and* $a_3 - a_2 = 19 - 21 = -2$

common difference is constant so it forms an $AP$

Let the number of rows in the flower bed be n.

Then $a = 23, d = -2, a_n = 5$ As, $a_n = a + (n - 1)d$

We have, $5 = 23 + (n - 1)(-2) \Longrightarrow -18 = (n - 1)(-2) \Longrightarrow 9 = n - 1 \Longrightarrow n = 9 + 1 \Longrightarrow n = 10$

So, there are 10 rows in the flower bed.

2.What is the $n^{th}$ term of AP? Check $-150$ is a term of the A.P : 11, 8, 5, 2, ..........

Solution: Given 11,8,5,2 are in AP

First term $a$ =1 Common difference d = 8 −11 = −3

$a_n = a + (n-1)d$ $i.e, a + (n-1)d = -150 \Longrightarrow 11 + (n-1)(-3) = -150$

$\Longrightarrow (n-1)(-3) = -150 - 11 \Longrightarrow (n-1)(-3) = -161$

$\Rightarrow (n-1) = \frac{-161}{-3} \Longrightarrow (n-1) = \frac{161}{3} \Longrightarrow n = \frac{161}{3} + 1 = \frac{161+3}{3} = \frac{164}{3}$

$n = \frac{164}{3}$ which is a fraction. n should be positive integer and not a fraction

So $-150$ is not a term of the A.P

3.Write the formulas for the following AP : $a_1, a_2, a_3, a_n$

A)$n^{th}$ term $a_n$ : $a_n = a + (n-1)d$

B)Sum of first n terms ($S_n$) : Sum of n terms of AP $S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of n terms of AP Whose first term 'a' and last term 'l' $S_n = \frac{n}{2}(a + l)$

C)Common difference

common difference $d = a_n - a_{n-1}$

D)If first term is a, last term is $a_n$ sum of first n terms ($S_n$)

Sum of n terms of AP Whose first term 'a' and last term 'l' $= \frac{n}{2}(a + l)$

4.How many terms of AP 24, 21, 18,..............must be taken so that their sum is 78 ?

Solution: Given 24, 21, 18....... are in AP

Here, a = 24, d = 21 – 24 = –3, Sn = 78. We need to find n.

We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$$78 = \frac{n}{2}[2 \times 24 + (n-1)(-3)] = \frac{n}{2}[48 - 3n + 3)] = \frac{n}{2}[51 - 3n]$$

$$2 \times 78 = n[51 - 3n] = 51n - 3n^2 \Rightarrow 156 = 51n - 3n^2 \Rightarrow 3n^2 - 51n + 156 = 0$$

$$\Rightarrow n^2 - 17n + 52 = 0 \Rightarrow n^2 - 13n - 4n + 52 = 0 \Rightarrow n(n-13) - 4(n-13) = 0$$

$$\Rightarrow (n-4)(n-13) = 0 \Rightarrow n - 4 = 0 \Rightarrow n = 4 \text{ or } n - 13 = 0 \Rightarrow n = 13$$

Both values of n are admissible.

So, the number of terms is either 4 or 13

5.How many multiples of 4 lie between 10 and 250 ?

Solution: By observation first multiple of 4 that is greater than 10 is 12 next will be 16 and soon ..

Hence the series will be as follows : 12,16,20,…… and the largest multiple between 4 and 250 is 248

( since we divide 250 by 2 then the remainder will be 2 250 – 2 = 248 )

The series : 12,16,20,……248 here $a = 12$ , $d = 4$

Let the $n^{th}$ term of AP $a_n = 248$

Using $a_n = a + (n-1)d$ then $248 = 12 + (n-1)4 \Rightarrow 248 - 12 = (n-1)4$

$$\Rightarrow 236 = (n-1)4 \Rightarrow \frac{236}{4} = n - 1$$

$$\Rightarrow 59 = n - 1 \Rightarrow n = 59 + 1 = 60$$

Answer: Therefore, there are 60 multiples of 4 between 10 and 250.

6.What is the formula for sum of first n terms if first and last terms are given. Find the sum of first 40 positive integers divisible by 6.

Solution: The positive integers that are divivsible by 6 are 6,12,18,..... 240

Hence the series will be as follows : 6,12,18,..... 240 in AP

First term $a = 6$ ,

last term $l = 240$ ,

number of terms n = 40

Sum of n terms of AP Whose first term $a$ and last term $l$ : $S_n = \frac{n}{2}(a + l)$

sum of first 40 positive integers divisible by 6:

$$S_{40} = \frac{40}{2}(6 + 240) \Rightarrow S_{40} = 20(246) = 4920$$

$$\therefore S_{40} = 4920$$

7.What is the $n^{th}$ term of A.P. Find the $20^{th}$ term from the end of A.P. 3, 8, 13,……… 253

Solution : Given AP : 3, 8, 13,…..253

Considered taking reverse AP : 253,…..13,8,3

Here, $a = 253$, $d = 3 - 8 = -5$ for finding $20^{th}$ term

we use $n^{th}$ term of an $AP$: $a_n = a + (n - 1)d$

$20^{th}$ term $a_{20} = 253 + (20 - 1)(-5) \Rightarrow a_{20} = 253 + (19)(-5) \Rightarrow a_{20} = 253 - 95 = 158$

∴ $20^{th}$ term from the end of given series = 158

## LEVEL-2 : SHINING STAR

### 8 MARKS QUESTION ANSWERS

1.A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year.

Find i)The production in the first year

ii)Find the production in the tenth year.

iii)Total production in first 7 years.

Solution : (i)Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1st , 2nd, 3rd, . . . , years will form an AP.

Let us denote the number of TV sets manufactured in the nth year by $a_n$ .

$Then, a_3 = 600$ $and$ $a_7 = 700$

$a_3 = 600 \Rightarrow a + 2d = 600 \ldots\ldots (i)$ $and$

$and$ $a_7 = 700 \Rightarrow a + 6d = 700 \ldots\ldots (ii)$ Solving these equations,

$eq\ (i) - eq\ (ii)$ $then$ ,

$$\begin{array}{r} a + 2d = 600 \\ - \; a + 6d = 700 \\ \hline -4d = -100 \end{array} \Rightarrow d = \frac{-100}{-4} = 25$$

$d = 25$ now substitute d value in eq (i)

then $a + 2 \times 25 = 600 \Rightarrow a + 50 = 600 \Rightarrow a = 600 - 50 = 550$

we get $d = 25$ and $a = 550$.

Therefore, production of TV sets in the first year = 550.

$(ii) Now\ a_{10} = a + 9d$

$= 550 + 9 \times 25$

$= 550 + 225 = 775$

So, production of TV sets in the 10th year is 775.

$(iii)$We know that $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2 \times 550 + (7-1)(25)] \Rightarrow S_7 = \frac{7}{2}[1100 + (6)(25)]$

$= \frac{7}{2}[1100 + 150] = \frac{7}{2}[1250] = 7 \times 625 = 4375$

Thus, the total production of TV sets in first 7 years is 4375.

2.In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be same as the class, in which they are studying e.g. a section of class I will plant one tree, a section of class II will plant 2 trees and soon till class XII. There are three sections of each class. How many trees will be planted by the students ?

Solution :

| | Class I | Class II | Class III | .................. | ClassXII |
|---|---|---|---|---|---|
| Section A | 1 | 2 | 3 | .................. | 12 |
| Section A | 1 | 2 | 3 | .................. | 12 |
| Section | 1 | 2 | 3 | .................. | 12 |
| Total | 3 | 6 | 9 | .................. | 36 |

We clearly observed that the number of trees planted by the students 3,6,9,........36 are in AP

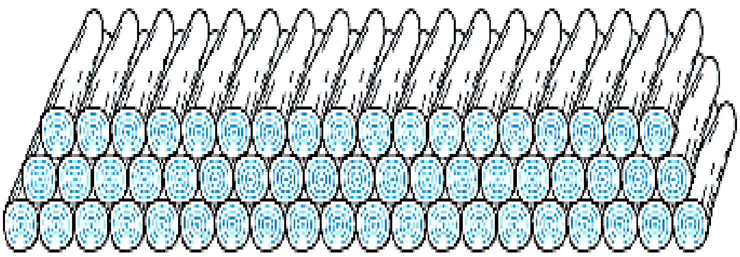
Here first term $a = 3$ ,
last term $l = 36$ ,
number of terms n = 12

Sum of n terms of AP Whose first term $a$ and last term $l$ : $S_n = \frac{n}{2}(a + l)$

$$S_{12} = \frac{12}{2}(3 + 36) = 6 \times 39 = 234$$

∴ The total number of trees will be planted by the students = 234

3.200 logs are stacked in the following manner 20 logs in the bottom row, 19 in the next row, 18 in the next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Solution: It can be observed that the numbers of logs in rows are in an A.P. 20, 19, 18....

First terms, a = 20

Common difference, $d = a_2 - a_1 = 19-20 = -1$

Sum of the n terms, $S_1 = 200$

We know that sum of n terms of AP.

$S_n = \frac{n}{2}[2a + (n-1)d] \Longrightarrow 200 = \frac{n}{2}[2 \times 20 + (n-1)(-1)] \Longrightarrow 400 = n[40 - n + 1]$

$\Longrightarrow 400 = n[41 - n] \Longrightarrow 400 = 41n - n^2$

$\Longrightarrow n^2 - 41n + 400 = 0$

$\Rightarrow n^2 - 16n - 25n + 400 = 0 \Rightarrow n(n-16) - 25(n-16) = 0 \Rightarrow (n-16)(n-25) = 0$
$\Rightarrow (n-16) = 0$ or $(n-25) = 0$

If $n - 16 = 0 \Rightarrow n = 16$ or If $n - 25 = 0 \Rightarrow n = 25$

$n^{th}$ term of an AP $a_n = a + (n-1)d$

$16^{th}$ term of an AP $a_{16} = [20 + (16-1) \times (-1)]$

$\Rightarrow a_{16} = [20 + 15 \times (-1)] \Rightarrow a_{16} = 20 - 15 = 5$

Similarly, $25^{th}$ term of an AP $a_{25} = [20 + (25-1) \times (-1)]$

$\Rightarrow a_{25} = [20 + 24 \times (-1)] \Rightarrow a_{16} = 20 - 24 = -4$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative 4, which is not possible.

∴ 200 logs can be placed in 16 rows.

The number of logs in the top (16th) row is 5.

## 4 MARKS QUESTION ANSWERS

1.If the sum of the first 14 terms of an AP is 1050 and its first term is 10, then find the $20^{th}$ term.

Solution : Here, $S_{14} = 1050$, $n = 14, a = 10$.

As $S_n = \frac{n}{2}[2a + (n-1)d] \Rightarrow S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d] \Rightarrow 1050 = 7[20 + 13d]$

$\Rightarrow 1050 = 140 + 91d \Rightarrow 91d = 1050 - 140 = 910 \Rightarrow d = \frac{910}{91} = 10$

Therefore, $a_{20} = 10 + (20 - 1) \times 10 = 200$, *i.e.* 20th term is 200.

2)What is the formula for sum of first n terms ? Show that $a_1, a_2, a_3, \ldots\ldots a_n$ form an AP.

Where an is defined as $a_n = 3 + 4n$. Find the sum of first 15 terms.

Solution : The formula for sum of first n terms $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}(a + l)$

Given: $a_n = 3 + 4n$

$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$

$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$

It can be observed that common difference $d = a_2 - a_1$ *and* $a_3 - a_2 = 15 - 11 = 4$

i.e, $a_2 - a_1 = 4 = a_3 - a_2$ the difference of $a_n - a_{n-1}$ is constant

Therefore $a_1, a_2, a_3 \ldots\ldots a_n$ form an AP.

sum of first 15 terms:

15 th term $a_{15}=3+4\times 15$ $[[\because given\ a_n = 3 + 4n.]$

$$a_{15} = 3+60=63$$

sum of first 15 terms $S_n = \frac{15}{2}(7+63)$ $[[\because S_n = \frac{n}{2}(a+l)]$

$$S_{15}=\frac{15}{2}(70) = 15 \times 35 = 525$$

3) What is the formula for sum of first n terms ? Find the sum of the odd numbers between 0 and 50.

Solution: The formula for sum of first n terms: $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}(a+l)]$

The odd numbers between 0 and 50 are 1,3,5,.........49 Which is in AP

Here first term $a = 1$ , common difference $d = 2$

let last term $l\ or\ a_n = 49$ ,

$a_n = a + (n-1)d$

$49 = 1 + (n-1)2 \Rightarrow 49 = 1+2n-2$

$\Rightarrow 49 = 2n - 1 \Rightarrow 49+1 = 2n \Rightarrow 50 = 20 \Rightarrow n = 25$

Therefore number of terms n =25

Sum of n terms of AP Whose first term $a$ and last term $l : S_n = \frac{n}{2}(a+l)$

$$S_{25} = \frac{25}{2}(1+49) = \frac{25}{2}(50) = 25 \times 25 = 625$$

∴ The sum of the odd numbers between 0 and 50= 625

4)Given a = 3, n = 8, S = 192 find d.

Solution: Given a = 3, n = 8, S = 192

The formula for sum of first n terms: $S_n = \frac{n}{2}[2a + (n-1)d]$

$$192 = \frac{8}{2}[2 \times 3 + (8-1)d] \Rightarrow 49 = 2n - 1 \Rightarrow 192 = 4[6 + 7d]$$

$$\Rightarrow \frac{192}{4} = 6 + 7d \Rightarrow 48 = 6 + 7d \Rightarrow 48 - 6 = 7d$$

$$\Rightarrow 42 = 7d \Rightarrow d = \frac{42}{7} \Rightarrow d = 6$$

5)How many three digit numbers are divisible by 7 ?

Solution: The first three digit number divisible by 7 is 105

Next number= 105 +7= 112

Therefore the series becomes 105,112,119,.... In AP

when we divide 999 by 7 the remainder will be 5 so 999-5= 994 is the greatest 3-digit number that is divisible by 7

first term a= 105 , common difference $d = 7$

let $a_n = 994$ , using $a_n = a + (n-1)d$

$994 = 105 + (n-1)7 \Longrightarrow 994 - 105 = (n-1)7$

$\Longrightarrow 889 = (n-1)7 \Longrightarrow \frac{889}{7} = n - 1 \Longrightarrow 127 = n - 1 \Longrightarrow 127 + 1 = n \Longrightarrow n = 128$

Therefore, three-digit numbers are divisible by 7 =128

6)Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution: Given second term $a_2 = 14$ and third term $a_3 = 18$

common difference $d = a_3 - a_2 = 18 - 14 = 4$

$n^{th}$ term of an AP $a_n = a + (n-1)d$

second term $a_2 = 14 \Longrightarrow 14 = a + 4 \Longrightarrow 14 - 4 = a \Longrightarrow 10 = a$

$a = 10$ , d= 4

The formula for sum of first n terms of an AP: $S_n = \frac{n}{2}[2a + (n-1)d]$

the sum of first 51 terms of an AP $S_{51} = \frac{51}{2}[2 \times 10 + (51-1)4]$

$= \frac{51}{2}[20 + 50 \times 4] = \frac{51}{2}[20 + 200]$

$= \frac{51}{2} \times 220 = 51 \times 110 = 5610$

Therefore, the sum of first 51 terms of an AP = 5610

## 1 MARK QUESTION ANSWERS

1. What is the common difference of $\frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ ,?

$Common\ difference\ (d) = a_2 - a_1$

$Common\ difference\ (d) = \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$

2. What is the next term of the series $3, 3+\sqrt{2}\ ,\ 3+2\sqrt{2}\ \ldots\ldots$

The list of numbers is $3, 3+\sqrt{2}\ ,\ 3+2\sqrt{2}\ \ldots\ldots$

$Common\ difference\ (d) = a_2 - a_1$

$Common\ difference\ (d) = 3 + \sqrt{2} - 3 = \sqrt{2}$

$next\ term\ of\ the\ series\ fourth\ term = a + 3d = 3 + 3\sqrt{2}$

3. What is the common difference of $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots\ldots$

$Common\ difference\ (d) = a_2 - a_1$

$Common\ difference\ (d) = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

4. Find the value of x if 13, 3x + 4, 13 are in A.P.

In an arithmetic progression (A.P.), the difference between consecutive terms is constant. Therefore

$3x + 4 - 13 = 13 - (3x + 4)$

$\Longrightarrow 3x - 9 = 9 - 3x \Longrightarrow 3x + 3x = 9 + 9 \Longrightarrow 6x = 18 \Longrightarrow x = 3$

5. Find the sum of first 100 natural numbers.

The sum of first n positive integers = $\frac{n(n+1)}{2}$

The sum of first 100 natural numbers = $\frac{100(100+1)}{2} = 50 \times 101 = 5050$

6. What is the common difference of the AP x-y, x, x+y.

Common difference $(d) = a_2 - a_1$

Common difference $(d) = x - (x - y) = x - x + y = y$

7. What is the sum of first 'n' even natural numbers.

The sum of first n even natural numbers $= n(n + 1)$

8. What is the sum of first 'n' odd natural numbers.

The sum of first n odd natural numbers. $= n^2$

9. Find the sum of first 25 odd natural numbers.

The sum of first 25 odd natural numbers. $= 25^2 = 625$

10. Find sum of first 10 natural numbers is ……..

The sum of first n positive integers $= \frac{n(n+1)}{2}$

The sum of first 10 positive integers $= \frac{10(10+1)}{2} = 5 \times 11 = 55$

11. The common difference of AP 3, 1, −1, −3.. is ……….

$Common\ difference\ (d) = a_2 - a_1$

$Common\ difference\ (d) = 1 - 3 = -2$

12. The sum of first 'n' natural numbers is

The sum of first 'n' natural numbers is $= \frac{n(n+1)}{2}$

13. If $n^{th}$ term of an AP is 3+ 4n then its common difference is

Common difference $(d) = a_2 - a_1$

Common difference $(d) = (3 + 4 \times 2) - (3 + 4 \times 1) = (3 + 8) - (3 + 4) = 11 - 7 = 4$

14. If a, b, c are in AP then b = ………………………………. $b = \frac{a+c}{2}$

15.Find the common difference of $\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2} \ldots\ldots$

Common difference $(d) = a_2 - a_1$

Common difference $(d) = -\frac{1}{2} - \frac{1}{2} = \frac{-1-1}{2} = \frac{-2}{2} = -1$

16.Determine the common difference of $\sqrt{27}$ , $\sqrt{108}$ , $\sqrt{243}$ , …

Common difference $(d) = a_2 - a_1$

CCommon difference $(d) = \sqrt{108} - \sqrt{27} = 6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$

17.Find the sum of first 100 natural numbers.

The sum of first n positive integers $= \frac{n(n+1)}{2}$

The sum of first 100 natural numbers $= \frac{100(100+1)}{2} = 50 \times 101 = 5050$

18.If $a = \frac{n}{n+1}$ then find $a_{2025}$ $a_{2025} = \frac{2025}{2025+1} = \frac{2025}{2026}$

19.Find the sum of first 1000 positive integers.

The sum of first n positive integers $= \frac{n(n+1)}{2}$

The sum of first 1000 positive integers $= \frac{1000(1000+1)}{2} = 500 \times 1001 = 500500$

20.The $30^{th}$ term of AP 10, 7, 4, ............

$30^{th}$ $term\ of\ the\ AP = a + 29d = 10 + 29 \times (-3) = 10 - 87 = -77$

21.Match the following. ( C )

p)Sum of first 10 natural numbers i) 129

q)Sum of first 10 even numbers ii) 55

r)Sum of first 10 prime numbers iii) 110

A) p-i, q-i, r-iii B) p-ii, q-i, r-iii

✓C) p-ii, q-iii, r-i D) p-iii, q-i, r-ii

22.Assertion (A) : 5 is the common difference of AP 11, 16,21 ...................................... 101

Reason (R) : The common difference of an AP $d = a_n - a_{n-1}$

Now choose the correct answer. ( C )

A) A is true, R is false B) A is false, R is true

C) Both A and R are true and R is correct explanation of A✓

D)Both A and R are true But R is not correct explanation of A

23.Statement I : 2, 4, 6, 8 ........................................ is in AP

Statement II : 10, 10, 10,......................................... is not in AP ( B )

A) Both statements are true B) Statement I is true But Statement II is false✓

C)Statement I is false But Statement II is true D) Both Statements are false.

24.If 7 times the $7^{th}$ term of an AP is equal to 11 times its $11^{th}$ term then its $18^{th}$ term. (D )

A) 7 B) 11 C) 18 D) 0 ✓

25.Which of the following is not form an AP ? ( C )

A) 2x, 4x, 6x, 8x, ..... B) 5, 0, – 5, – 10, ...........

✓C) 25, 20, 17, .......... D) −3, – 6, – 9, ................

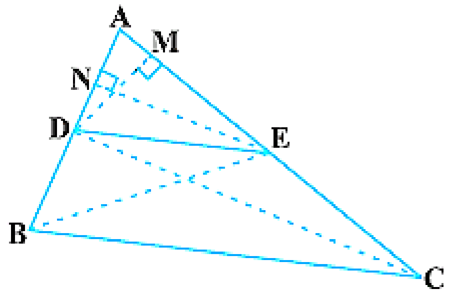
# 6. TRIANGLES

[11 Marks] [1 + 2 + 8 = 11 M]

## LEVEL-1: RISING STAR

### 8 MARKS QUESTION ANSWERS

1.State and prove Basic Proportionality Theorem




Basic Proportionality Theorem:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Proof: We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively

We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Now, ar(ADE) $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}\, AD \times EN.$

Similarly, ar(BDE) $= \frac{1}{2}\, DB \times EN,$

ar(ADE) $= \frac{1}{2} AE \times DM$ and ar(DEC) $= \frac{1}{2}\, EC \times DM.$

Therefore $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$ ……… (1)

and $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2}\, EC \times DM} = \frac{AE}{EC}$ …… (2)

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

So, ar(BDE) = ar(DEC) ……… (3)

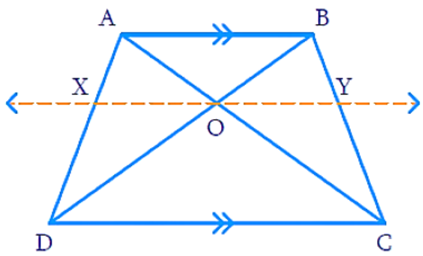
Therefore, from (1), (2) and (3)

we have $\frac{AD}{DB} = \frac{AE}{EC}$

Basic Proportionality Theorem is proved

2.ABCD is a trapezium in which AB// DC and its diagonals intersect each other at the point O.

Show that $\frac{AO}{CO} = \frac{BO}{DO}$




Solution: Given ABCD is a trapezium, AB||CD, AC and BD intersect at 'O'

RTP: $\frac{AO}{CO} = \frac{BO}{DO}$

Construct XY parallel to AB and CD (XY||AB, XY||CD) through 'O'

In ΔABC OY ∥ AB ( ∴ construction)

According to BPT $\frac{BY}{CY} = \frac{AO}{CO}$ ……… (i)

In ΔBCD OY ∥ CD ( ∴ construction)

According to BPT $\frac{BY}{CY} = \frac{BO}{DO}$ ……………. (ii)

From (i) and (ii) $\frac{AO}{CO} = \frac{BO}{DO}$

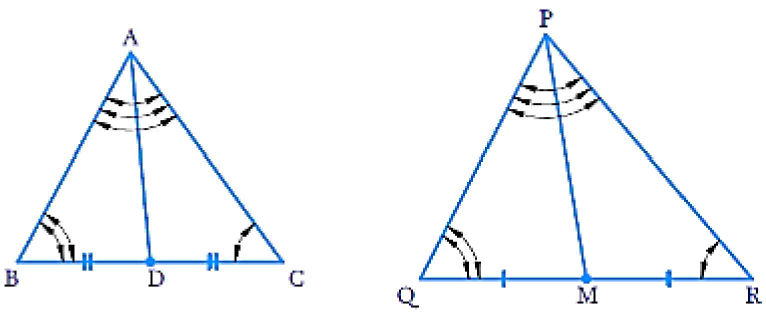
Hence $\frac{AO}{CO} = \frac{BO}{DO}$ is proved.

3.If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC is similar to ΔPQR.

Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Solution: We know if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This is referred as SAS criterion for two triangles.




Given, ΔABC ~ ΔPQR

$\Rightarrow \angle ABC = \angle PQR$ (corresponding angles) --------- (1)

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR}$ (∵corresponding sides)

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{BC}{2}}{\frac{QR}{2}}$$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ (D and M are mid-points of BC and QR) ------------ (2)

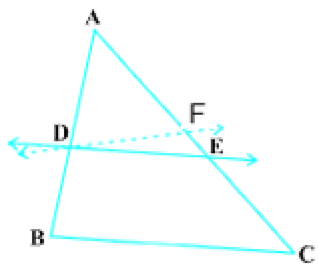
In ΔABD and ΔPQM, ∠ABD = ∠PQM (from 1)

$\frac{AB}{PQ} = \frac{BD}{QM}$ (from 2) $\Rightarrow$ ΔABD ~ ΔPQM (SAS criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (∵corresponding sides)

$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$ Hence proved.

4.Prove that if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.




Given: In ΔABC, line DE intersects AB at D and AC at E such that $\frac{AD}{DB} = \frac{AE}{EC}$

RTP: DE || BC (DE is parallel to BC)

Assumption (for contradiction): Assume DE is not parallel to BC

Construction: Draw a line DF parallel to BC, where F is on AC (so DF || BC)

From BPT: $\frac{AD}{DB} = \frac{AF}{FC}$ (DF || BC)

Compare Ratios: We are given $\frac{AD}{DB} = \frac{AE}{EC}$ and we derived $\frac{AD}{DB} = \frac{AF}{FC}$

Therefore, $\frac{AE}{EC} = \frac{AF}{FC}$

Deduction: If $\frac{AE}{EC} = \frac{AF}{FC}$ , then by adding 1 to both sides and simplifying

$$\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1 \Rightarrow \frac{AE+EC}{EC} = \frac{AF+FC}{FC} \Rightarrow \frac{AC}{EC} = \frac{AC}{FC}$$

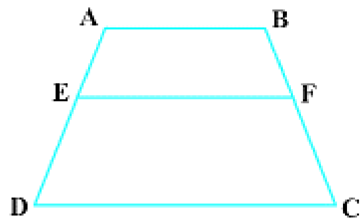
we find that point E and point F must coincide (E = F).

Conclusion: Since our constructed parallel line DF coincides with the given line DE (because E=F), the line DE must indeed be parallel to BC.

Hence, given 'If a line divides two sides of a triangle in the same ratio, it is parallel to the third side.' is proved

5.ACBD is a trapezium with AB//DC. E and Fare points on non-parallel sides AD and BC respectively.

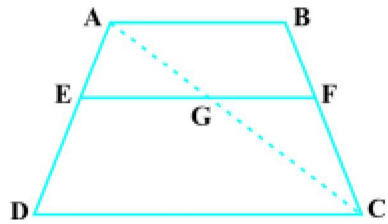
Such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$




Solution: Let us join AC to intersect EF at G

AB || DC and EF || AB (Given)

So, EF || DC (Lines parallel to the same line are parallel to each other)




Now, in $\Delta$ADC, EG || DC (As EF || DC)

So, $\frac{AE}{ED} = \frac{AG}{GC}$ ( From BPT) ---------1

Similarly, from $\Delta$CAB,

So, $\frac{CG}{AG} = \frac{CF}{BF}$ ( From BPT) $\frac{AG}{GC} = \frac{BF}{FC}$ --------2

Therefore, from (1) and (2), $\frac{AE}{ED} = \frac{BF}{FC}$

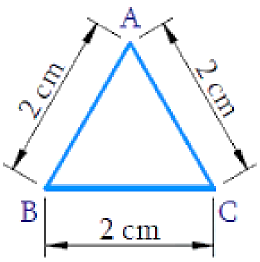
## 2 MARKS QUESTION ANSWERS

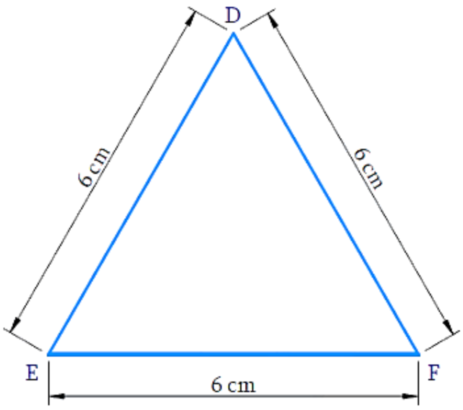
1.Give an example for (i) Similar figures (ii) Non-similar figures.

Solution:

(i) Similar figures

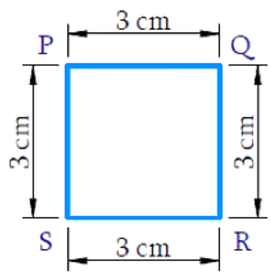
(a) Two equilateral triangles of sides 2cm and 6cm.

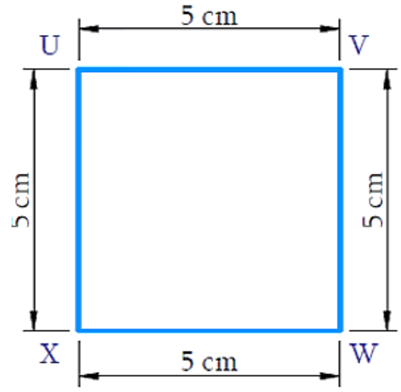



$\Delta$ ABC ~ $\Delta$ DEF

(b) Two squares of sides 3cm and 5cm.



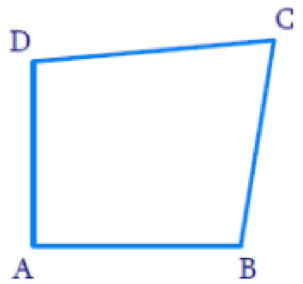



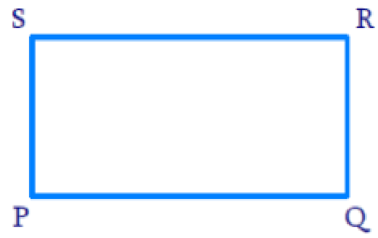

□PQRS ~ □UVWX

(ii) Non-similar figures.

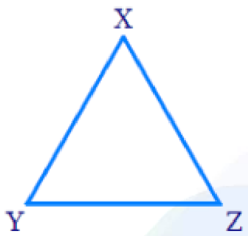
Solution :

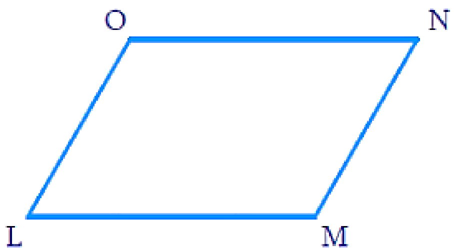
(a) A quadrilateral and a rectangle.




(b) A triangle and a parallelogram




2.What are the conditions for the similarity of triangles?

Two triangles are similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or proportion).

3.State S.A.S criteria in similarity of triangles.

S.A.S criteria: If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

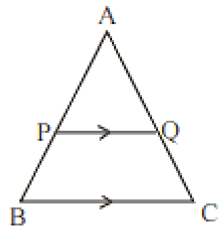
4.State A.A. criteria in similarity of triangles.

A.A. criteria: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar

5.State S.S.S criteria in similarity of triangles.

S.S.S criteria: If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar

6.In given figure, PQ//BC, PQ = 3 cm, BC = 9 cm and AC = 7.5 cm. Find the length of AQ?




Solution: Given that PQ∥BC and PQ = 3 cm, BC = 9 cm and AC = 7.5 cm.

$\Delta APQ \sim \Delta ABC$ (∵ from AA similarity criterion

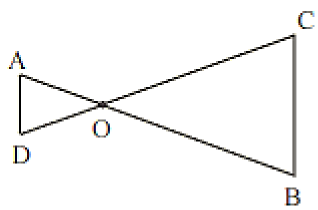
The ratio of corresponding sides in similar triangles is equal: $\frac{AQ}{AC} = \frac{PQ}{BC}$

Substitute the given values in above equation

$\frac{AQ}{7.5} = \frac{3}{9} \Rightarrow \frac{AQ}{7.5} = \frac{1}{3} \Rightarrow AQ = \frac{7.5}{3} = 2.5$

∴ The length of $AQ$= 2.5 cm

7. In given figure, OA. OB = OC. OD Show that ∠A = ∠C and ∠B = ∠D




Solution: OA. OB = OC. OD (Given)

So, $\frac{OA}{OC} = \frac{OD}{OB}$ ---------(1)

Also, we have ∠AOD = ∠ COB (Vertically opposite angles) …… (2)

Therefore, from (1) and (2), Δ AOD ~ Δ COB (∵ SAS similarity criterion)

So, ∠A = ∠ C and ∠ D = ∠ B (∵ Corresponding angles of similar triangles)

## LEVEL-2 : SHINING STAR

### 8 MARKS QUESTION ANSWERS

1.A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/sec. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Solution: Let AB denote the lamp-post and CD the girl after walking for 4 seconds away from the lamp-post

From the figure, you can see that DE is the shadow of the girl. Let DE be $x$ metres.

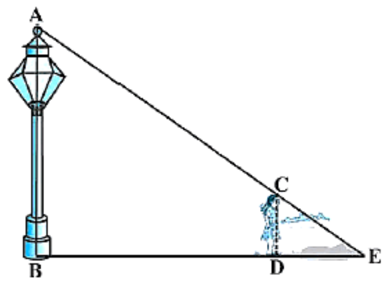
Now, BD = 1.2 m × 4 = 4.8 m. ( ∵ distance = speed × time )

In ΔABE and in ΔCDE

∠B = ∠D ( ∵ Each is of 90° because lamp-post as well as the girl are standing vertical to the ground)

and ∠E = ∠E ( ∵ common angle )

so, ΔABE ~ ΔCDE( ∵ AA similarity criterion )




therefore, $\frac{BE}{DE} = \frac{AB}{CD}$

1m = 100cm , 90cm = $\frac{90}{100}$ m = 0.9m

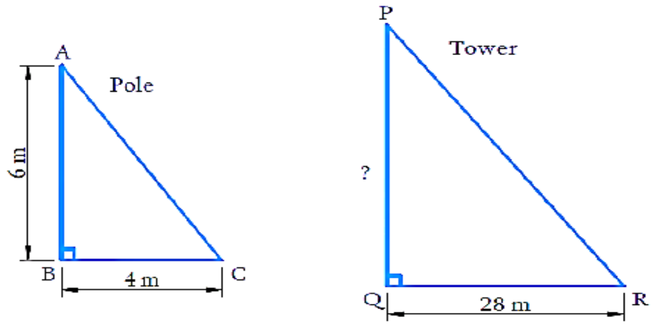
i.e. $\frac{4.8+x}{x} = \frac{3.6}{0.9}$

i.e. $4.8 + x = 4x$

$\Rightarrow 4x - x = 4.8 \Rightarrow 3x = 4.8 \Rightarrow x = 1.6$m

∴ The length of her shadow after 4 seconds =1.6 m

2.A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.




Solution: AB is the length of the pole = 6m

BC is the shadow of pole = 4m

PQ is the tower =?

QR is the shadow of the tower = 28m

In $\Delta ABC$ and in $\Delta PQR$

$\angle B = \angle Q = 90°$ ( ∵ The objects and shadows are perpendicular to each other)

$\angle A = \angle P$ (∵ Sunray fall on the pole and tower at the same angle, at the same time)

$\Longrightarrow \Delta ABC \sim \Delta PQR$ ( ∵ AA similarity criterion )

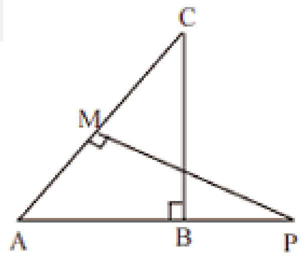
The ratio of any two corresponding sides in two equiangular triangles is always the same.

$$\Longrightarrow \frac{AB}{BC} = \frac{PQ}{QR} \Longrightarrow \frac{6m}{4m} = \frac{PQ}{28m} \Longrightarrow PQ = \frac{6\times28m}{4} = 42m$$

Hence the height of the tower= 42m

3.In given figure ABC and AMP are two right triangles, right angled at B and M respectively.

Prove that i) $\Delta ABC \sim \Delta AMP$ ii) $\frac{CA}{PA} = \frac{BC}{MP}$




Solution: In $\Delta ABC$ and $\Delta AMP$

$\angle ABC = \angle AMP = 90°$

$\angle BAC = \angle MAP$ (∵ Common angle)

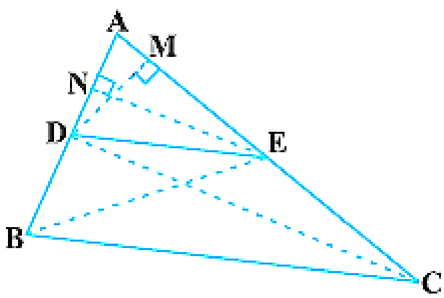
$\therefore \Delta ABC \sim \Delta AMP$

ii) As we know that the ratio of any two corresponding sides in two equiangular triangles is always the same

In two equiangular triangles $\Delta ABC$ and $\Delta AMP$

$\frac{CA}{PA} = \frac{BC}{MP}$ [ ∵ $\Delta ABC = \Delta AMP$]

4.If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.




Proof: We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively

We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$

Let us join BE and CD and then draw DM ⊥ AC and EN ⊥ AB.

Now, $ar(ADE) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AD \times EN.$

Similarly, $ar(BDE) = \frac{1}{2} DB \times EN,$

$ar(ADE) = \frac{1}{2} \times AE \times DM$ and $ar(DEC) = \frac{1}{2} \times EC \text{ x } DM.$

$\therefore \frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$ ......... (1)

and $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$ ..............(2)

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

So, ar(BDE) = ar(DEC) _______________ ( 3)

Therefore, from (1), (2) and (3)

we have $\frac{AD}{DB} = \frac{AE}{EC}$

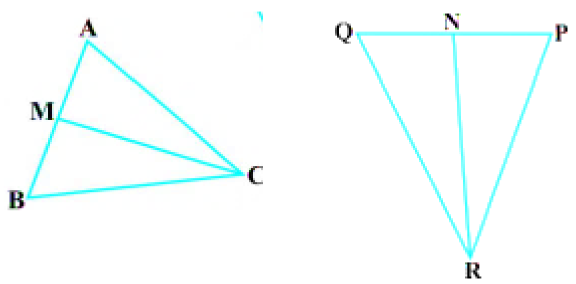
Hence the Theorem is proved.

5. CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC ~ ΔPQR.

Prove that i) ΔAMC ~ ΔPNR

ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

iii) ΔCMB ~ ΔRNQ




Solution:

i) ΔABC ~ ΔPQR (Given)

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ... ... ... ( 1)

and ∠A = ∠P , ∠B = ∠Q , ∠C = ∠R ... ... ... ( 2)

But AB = 2 AM and PQ = 2 PN (As CM and RN are medians)

As From (1) $\frac{2AM}{2PN} = \frac{CA}{RP} \Rightarrow \frac{AM}{PN} = \frac{CA}{RP}$ ... ... ... ( 3 )

Also, ∠MAC = ∠NPR [From (2)] ----------(4)

So, from (3) and (4), ΔAMC ~ ΔPNR (SAS similarity) -----------(5)

ii) $\frac{CM}{RN} = \frac{CA}{RP}$ [ $\because$ $\Delta AMC \sim \Delta PNR$ ] ... ... ... ( 6 )

but from ( 1 ) $\frac{CA}{RP} = \frac{AB}{PQ}$ ... ... ... ( 7 )

$\therefore \frac{CM}{RN} = \frac{AB}{PQ}$ [From (6), (7) ] ... ... ... ... ... ( 8 )

iii) Again from ( 1 ) $\frac{AB}{PQ} = \frac{BC}{QR}$

from (8) $\frac{CM}{RN} = \frac{BC}{QR}$ ... ... ... . . (9)

Also $\frac{CM}{RN} = \frac{AB}{PQ} = \frac{2BM}{2QN}$

$\therefore \frac{CM}{RN} = \frac{BM}{QN}$ --------- (10)

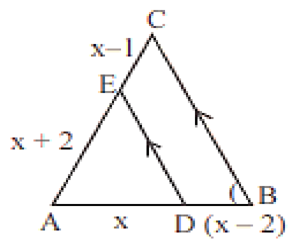
$\Longrightarrow \frac{CM}{RN} = \frac{BC}{QR} = \frac{BM}{QN}$ [$\because$ from (9) and (10)]

$\therefore \Delta CMB \sim \Delta RNQ$ ( $\because$ SSS criterion )

## 2MARKS QUESTION ANSWERS

1.In the figure, DE//BC, if AD = x, DB = x − 2, AE = x + 2 and EC = x − 1.

Find the value of x.




Solution: $\frac{AD}{DB} = \frac{AE}{EC}$ [ $\because$ DE ∥ BC from BPT]

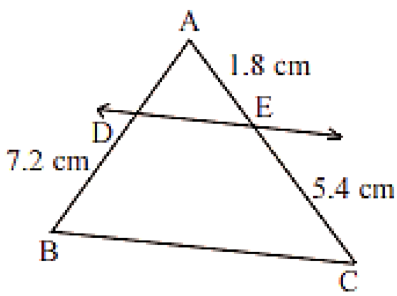
$\Longrightarrow \frac{x}{x-2} = \frac{x+2}{x-1} \Rightarrow x\,(x-1) = (x+2)(x-2)$

$\Longrightarrow x^2 - x = x^2 - 4$

$\Longrightarrow -x = -4 \Longrightarrow x = 4$

$\therefore x = 4$

2.In the figure, DE//BC. Find AD.




Solution: from figure **DB = 7.2cm, AE = 1.8cm, EC = 5.4cm**

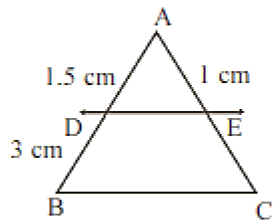
$\frac{AD}{DB} = \frac{AE}{EC}$ [ $\because$ DE∥ BC from BPT]

$\frac{AD}{7.2} = \frac{1.8}{5.4} \Longrightarrow \frac{AD}{7.2} = \frac{1}{3} \Longrightarrow AD = \frac{7.2}{3} = 2.4cm$

$\therefore$AD = 2.4 cm

## 1 MARK QUESTION ANSWERS

1. In ΔABC, DE//BC then length of EC = (A)




A)2 cm B) 2.25 cm C) 3.5 cm D) 1.4 cm

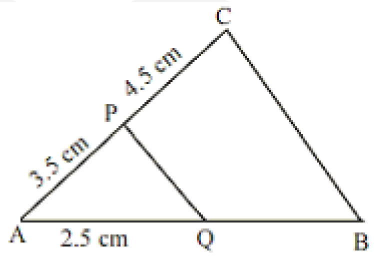
Solution: from figure $\mathbf{AD = 3cm}$, $\mathbf{DB = 3cm}$ , $\mathbf{AE = 1cm}$

$\frac{AD}{DB} = \frac{AE}{EC}$ [ ∵ DE∥ BC from BPT]

$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow 0.5 = \frac{1}{EC} \Rightarrow \frac{5}{10} = \frac{1}{EC} \Rightarrow \frac{1}{2} = \frac{1}{EC} \Rightarrow EC = 2cm$

$\therefore EC = 2cm$

2.In the given figure, PQ//CB, then QB = .................. (C)




A)2.4 cm B) 2.7 cm C) 3.2 cm D) 4.2 cm

Solution: from figure AP = 3.5cm, PC = 4.5cm, AQ = 2.5cm

$\frac{AP}{PC} = \frac{AQ}{QB}$ [ ∵ PQ∥ CB from BPT]

$\frac{3.5}{4.5} = \frac{2.5}{QB} \Longrightarrow \frac{7}{9} = \frac{2.5}{QB} \Longrightarrow QB = \frac{2.5 \text{ x } 9}{7} = \frac{22.5}{7} cm = 3.2cm$

$\therefore QB = 3.2cm$

3.Write the statement of Basic proportional theorem. (Thale's Theorem)

Basic Proportionality Theorem:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

4.Statement I: All squares are congruent to each other. ( B )

Statement II: All circles are similar

A) Statement I is true, Statement II is false. B) Statement I is false, Statement II is true

C) Both Statements are true. D)Both Statements are false.

5.Statement I : All similar triangles are congruent. ( C )

Statement II : All right angled isosceles triangles are similar.

A. Both Statements are true. B. Statement I is true and Statement II is false.

C. Statement I is false and Statement II is true D. Both Statements are false.

6. If ΔABC and ΔPQR, if $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then which of the following is true? ( D)

A) ΔCBA ~ ΔPQR B) ΔPQR ~ ΔABC C) ΔBCA ~ ΔPQR D) ΔPQR ~ ΔCAB

7.Which of the following is not a similarity criterion for two triangles ( C )

A) S.A.S B) A.A.A C) A.S.A D) S.S.S

8.If ΔABC ~ ΔXYZ then which of the following is true? (No answer)

A) $\angle A = \angle Z$ B) $\frac{AC}{BC} = \frac{YZ}{XZ}$ C) $\angle B = \angle Z$ D) $BC = YZ$

Analyzing options:

A) $\angle A=\angle Z$: This is incorrect.

$\angle A$ must equal to $\angle X$

B): $\frac{\text{AC}}{\text{BC}} = \frac{\text{YZ}}{\text{XZ}}$ This is incorrect. The correct ratio pairing would be $\frac{\text{AC}}{\text{BC}} = \frac{\text{XZ}}{\text{YZ}}$

C) $\angle B=\angle Z$: This is incorrect. $\angle B$ must equal to ∠Y

D) $BC=YZ$: This is incorrect. The sides are in proportion, not necessarily equal in length.

Equality($BC=YZ$) only occurs if the triangles are also congruent.

Therefore, none of the options listed are universally true for all similar triangles

9. Which of the following statement is true? ( D )

A) All equilateral triangles are similar B) All circles are similar

C) All squares are similar D) All the above

10. ΔABC ~ ΔPQR and ∠A = 70° then ∠Q + ∠R = ............ ( C )

A)180° B) 120° C) 110° D) 70°

11. ΔABC and ΔDEF are similar. ∠A = 40° and ∠F = 70°, then the value of ∠B + ∠E is ( B )

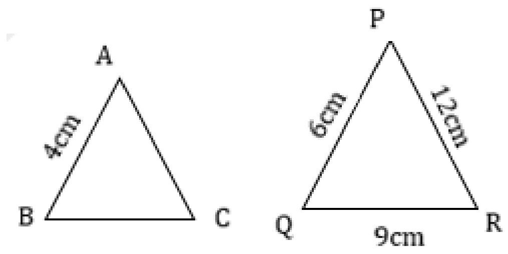
A)110° B) 140° C) 100° D) 70°

12. ΔABC and ΔPQR are similar. If AB = 4 cm, PQ = 6cm, QR = 9 cm and PR = 12 cm

then the perimeter of ΔABC is .......... cm. ( C )

A) 15 cm B) 20 c m C) 18 cm D) 24 cm

Solution: Given ΔABC ~ ΔPQR ,




AB = 4 cm, PQ = 6cm, QR = 9 cm and PR = 12 cm

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

$$\frac{AB}{PQ} = \frac{BC}{QR} \Longrightarrow \frac{4}{6} = \frac{BC}{9} \Longrightarrow BC = \frac{9 \times 4}{6}\ cm = 6cm$$

$$\frac{AB}{PQ} = \frac{CA}{RP} \Longrightarrow \frac{4}{6} = \frac{CA}{12} \Longrightarrow CA = \frac{12 \times 4}{6}\ cm = 8cm$$

The perimeter of ΔABC = AB + BC + CA = 4 cm + 6 cm + 8 cm= 18cm

$$\frac{AB}{PQ}\{PQ + QR + RP\} = \frac{4}{6}\{6cm + 9cm + 12cm\} = \frac{2}{3}\{27cm\} = 18cm$$

13.Statement I : Two equilateral triangles are always similar.

Statement II : ΔABC ~ ΔXYZ then AB : XY = AC : XZ ( A )

A) Both Statements are true. B) Statement I is true and Statement II is false.

C)Statement I is false and Statement II is true D) Both Statements are false.

14.Two triangles with equal corresponding sides always similar. Is it true or false? True

Two triangles with equal corresponding sides are always similar by SSS criterion.

15.Name the two figures that are always similar. i)circles ii) equilateral triangles

The two types of figures that are always similar are circles and equilateral triangles (or any regular polygons like squares)

16.If ΔABC and ΔDEF, if $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar when ( D )

A) ∠B = ∠E B) ∠A = ∠F C) ∠A = ∠D D) ∠B = ∠D

$$\frac{AB}{DE} = \frac{BC}{FD} \Rightarrow \frac{AB}{BC} = \frac{DE}{FD} \Rightarrow \frac{A}{C} = \frac{E}{F}\ \ \textbf{so}\ \ \angle A = \angle E, \angle C = \angle F \text{ and } \angle B = \angle D$$

17.Assertion (A) **:** D and E are the points of sides AB and AC of $\Delta ABC$ respectively and

$DE//BC$ then $AD : DB = AE : EC$

Reason (R):If a line is drawn parallel to one side of a triangle, it will divide the other two sides in the same ratio. ( A )

A) Both Assertion and Reason are true and reason is the correct explanation of Assertion.

B) Both Assertion and Reason are true but reason is not the correct explanation of Assertion.

C)Assertion is true but Reason is false.

D)Assertion is false but Reason is true.

18. $\Delta ABC \sim \Delta PQR$, $\angle A = 50°$, $\angle Q = 40°$ then $\Delta QPR$ is (B )

A) Isosceles triangle

B) Right angled triangle

C) Equilateral triangle

D) Isosceles Right-angled triangle

$\angle A + \angle B + \angle C = 180° = \angle P + \angle Q + \angle R$

$\Rightarrow 50° + 40° + \angle R = 180° = 90° + \angle R = 180° \Rightarrow \angle R = 90°$ $\Delta QPR$ is a Right-angled triangle

19)Statement A**:** All circles are similar. ( D )

Statement B**:** All triangles are similar.

A) Both A and B are true

B) Both A and B are false

C) A is false B true

D) A true B false

20.Write the proportionality relation if MN//BC in $\Delta ABC$

According to the Basic Proportionality Theorem

If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\text{So } \frac{AM}{MB} = \frac{AN}{NC}$$

# 7 COORDINATE GEOMETRY

**[10 Marks] [ 2 + 8 = 10 M]**

## LEVEL-1: RISING STAR

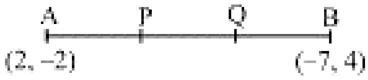
### 8 MARKS QUESTION ANSWERS

1.Two students claim to have found the points of trisection of the line segment joining

A (2, − 2) and B (− 7, 4) as follows:

Bharath: (−1, 0) and (− 4,2). Vinod: (1, 0) and (4, −2). Who is correct? Justify?

Solution: Let P and Q be the points of trisection of AB i.e., AP = PQ = QB




Therefore, P divides AB internally in the ratio 1: 2. Therefore, the coordinates of P, by applying the section formula, are

$$P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$P(x,y) = \left(\frac{1\times(-7)+2\times(2)}{1+2}, \frac{1\times(4)+2\times(-2)}{1+2}\right) = \left(\frac{-7+4}{3}, \frac{4-4}{3}\right) = \left(\frac{-3}{3}, \frac{0}{3}\right) = (-1, 0)$$

∴ The co-ordinates of point P are (−1, 0)

Now, Q also divides AB internally in the ratio 2: 1. So, the coordinates of Q are

$$Q(x,y) = \left(\frac{2\times(-7)+1\times(2)}{2+1}, \frac{2\times(4)+1\times(-2)}{2+1}\right) = \left(\frac{-14+2}{3}, \frac{8-2}{3}\right) = \left(\frac{-12}{3}, \frac{6}{3}\right) = (-4, 2)$$

Therefore, the coordinates of the points of trisection of the line segment joining

A and B are P (– 1, 0) and Q (– 4, 2). So Bharath: (−1, 0) and (− 4,2) is correct

2.Do the points (3, 2), (−2, −3) and (2, 3) form a triangle? If so, name the type of triangle formed:

Solution: Let us apply the distance formula to find the distances PQ, QR and PR,

where P (3, 2), Q (−2, −3) and R (2, 3) are the given points.

Using distance between two points $(x_1, y_1), (x_2, y_2)$ formula $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$PQ = \sqrt{(-2-3))^2 + (-3-2)^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50}$ units

$= \sqrt{2\times 25} = 5\sqrt{2} = 5 \times 1.414 = 7.07$ (approx.)

$(PQ)^2 = (\sqrt{50})^2 = 50$ units

$QR = \sqrt{(2-(-2))^2 + (3-(-3))^2} = \sqrt{(2+2))^2 + (3+3))^2} = \sqrt{(4)^2 + (6)^2} = \sqrt{16+36} = \sqrt{52}$ units

$= \sqrt{4\times 13} = 2\sqrt{13} = 2 \times 3.605 = = 7.21$ (approx.)

$(QR)^2 = (\sqrt{52})^2 = 52$ units

$PR = \sqrt{(2-3)^2 + (3-2)^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$ units = 1.41 (approx.)

$(PR)^2 = (\sqrt{2})^2 = 2$ units

Since the sum of any two of these distances is greater than the third distance,

therefore, the points P, Q and R form a triangle.

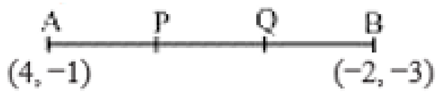
Also, $PQ^2 + PR^2 =$ 50 units + 2 units = 52 units = $QR^2$

$PQ^2 + PR^2 = QR^2$

by the converse of Pythagoras theorem, we have $\angle P = 90°$.

∴PQR is a right triangle.

3.Two students Ramu and Somu solved the problem of finding trisection points of the line segment joining A (4, −1) and B (−2, −3). Ramu got the points as $(2, -\frac{5}{3})$ and $(0, -\frac{7}{3})$ while somu got (2, −2) and (0, −3) Evaluate and decide which one is correct.




Solution: Let P and Q be the points of trisection of AB i.e., AP = PQ = QB

Therefore, P divides AB internally in the ratio 1: 2. Therefore, the coordinates of P,

Using Section formula $P(x, y) = (\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n})$

$$P(x, y) = \left(\frac{1\times(-2) + 2\times(4)}{1+2}, \frac{1\times(-3) + 2\times(-1)}{1+2}\right) = \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right) = \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, \frac{-5}{3}\right)$$

∴ The co-ordinates of point P are $\left(2, \frac{-5}{3}\right)$

Now, Q also divides AB internally in the ratio 2: 1. So, the coordinates of Q are

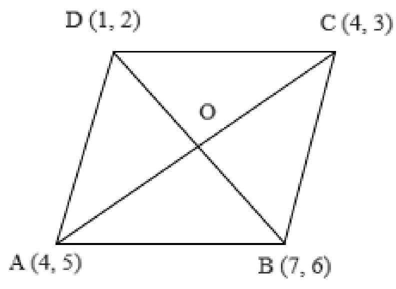
$$Q(x, y) = \left(\frac{2\times(-2) + 1\times(4)}{2+1}, \frac{2\times(-3) + 1\times(-1)}{2+1}\right) = \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right) = \left(\frac{0}{3}, \frac{-7}{3}\right) = \left(0, \frac{-7}{3}\right)$$

Therefore, the coordinates of the points of trisection of the line segment joining

A and B are $(2, -\frac{5}{3})$ and $(0, -\frac{7}{3})$ So Ramu is correct

4.Check whether the points (4, 5), (7, 6), (4, 3) and (1, 2) taken in order form a parallelogram?

Solution: Let A (4, 5), B (7, 6), C (4, 3) and D (1, 2) be the given points.




Using distance between two points $(x_1, y_1), (x_2, y_2)$:

formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$ units

$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$ units

$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$ units

$DA = \sqrt{(4-1)^2 + (5-2))^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$ units

$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{(0)^2 + (-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$ units

$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} = \sqrt{4 \times 13} = 2\sqrt{13}$ units

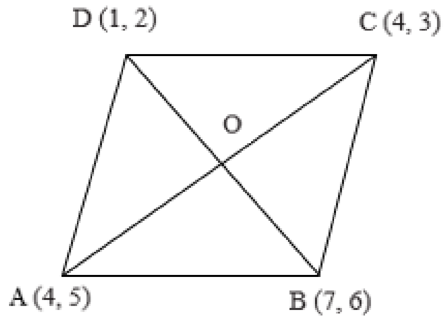
Since, AB = BC and CD = DA and AC ≠ BD,

We clearly observe that opposite sides are equal and its diagonals are not equal.

Therefore, ABCD quadrilateral is a parallelogram.

Alternate method:

Solution: Let A (4, 5), B (7, 6), C (4, 3) and D (1, 2) be the given points.




Using midpoint formula $P(x, y) = \left(\frac{x_1 + x_2}{2}\ ,\ \frac{y_1 + y_2}{2}\right)$

mid-point of AC $= \left(\frac{4+4}{2}, \frac{5+3}{2}\right) = \left(\frac{8}{2}, \frac{8}{2}\right) = (4, 4)$

mid-point of BD $= \left(\frac{7+1}{2}, \frac{6+2}{2}\right) = \left(\frac{8}{2}, \frac{8}{2}\right) = (4, 4)$

coordinates of the mid-point of AC = coordinates of the mid-point of BD

We know that diagonals of a parallelogram bisect each other.

∴ ABCD quadrilateral is a parallelogram.

5.Verify whether the points (3, 0), (4, 5), (−1, 4) and (−2, −1) are vertices of rhombus? If so, find the area of rhombus

Solution: Let A (3, 0), B (4, 5), C (−1, 4) and D (−2, −1) be the given points.

Using distance between two points $(x_1, y_1), (x_2, y_2)$ formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$AB = \sqrt{(4 - 3)^2 + (5 - 0)^2} = \sqrt{(1)^2 + (5)^2} = \sqrt{1 + 25} = \sqrt{26}$ units

$BC = \sqrt{(-1 - 4)^2 + (4 - 5)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26}$ units

$CD = \sqrt{(-2 - (-1))^2 + (-1 - 4)^2} = \sqrt{(-2 + 1)^2 + (-5)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1 + 25} = \sqrt{26}$ units

$DA = \sqrt{(3 - (-2))^2 + (0 - (-1))^2} = \sqrt{(3 + 2)^2 + (0 + 1)^2} = \sqrt{(5)^2 + (1)^2} = \sqrt{25 + 1} = \sqrt{26}$ units

$AC = \sqrt{(-1 - 3)^2 + (4 - 0)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16 + 16} = \sqrt{32} = \sqrt{2 \times 16} = 4\sqrt{2}$ units

$BD = \sqrt{(-2 - 4)^2 + (-1 - 5)^2} = \sqrt{(-6)^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{2 \times 36} = 6\sqrt{2}$ units

Since, AB = BC = CD = DA and AC ≠ BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are not equal.

Therefore, ABCD is a rhombus.

Area of the rhombus ABCD $= \frac{1}{2}$ (Product of lengths of diagonals)

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= \frac{1}{2} \times 24 \times (\sqrt{2})^2$

$= \frac{1}{2} \times 24 \times 2$

$= 24$ sq. units

∴ Area of rhombus = 24 sq. units

## 2 MARKS QUESTION ANSWERS

1.Write the formula to find the distance between:

(a) the two points $(x_1, y_1)$, and $(x_2 , y_2 )$ (b) The origin and the point$(x, y)$.

(a) distance between two points $(x_1, y_1), (x_2, y_2)$ formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(b) distance between the origin and the point $(x, y)$ formula $= \sqrt{x^2 + y^2}$

2.Find the coordinates of mid-point on the line segment joining the points $(-2, -2)$ and $(2, -4)$.

midpoint formula $P(x, y) = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$

mid-point on the line segment joining the points $(-2, -2)$ and $(2, -4)$

$$P(x, y) = \left(\frac{-2+2}{2} , \frac{-2+(-4)}{2}\right) = \left(\frac{0}{2} , \frac{-2-4}{2}\right) = \left(\frac{0}{2} , \frac{-6}{2}\right) = (0, -3)$$

3.Find the distance between the points (2, 3) and (4, 1)

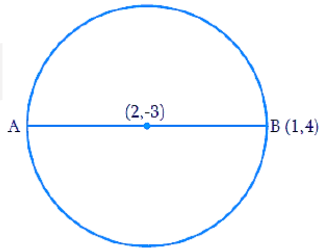
distance between two points $(x_1, y_1), (x_2, y_2)$ formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

the distance between the points (2, 3) and (4, 1) $= \sqrt{(4-2)^2 + (1-3))^2}$

$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4}$

$= \sqrt{2 \times 4} = 2\sqrt{2}$ units

4.Find the coordinates of the point A, where AB is the diameter of a circle whose center is $(2, -3)$ and B is $(1, 4)$.




Solution: Given, Let the coordinates of point A be $(x, y)$.

Midpoint formula $= \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$

Mid-point of AB is C $(2, -3)$, which is the center r of the circle.

$$C\,(2, -3) = \left(\frac{x+1}{2} , \frac{y+4}{2}\right) \Rightarrow \frac{x+1}{2} = 2 , \frac{y+4}{2} = -3$$

$$x + 1 = 4 \Rightarrow x = 4 - 1 \Rightarrow x = 3 ,$$

$$and \quad y + 4 = -6 \Rightarrow y = -6 - 4 \Rightarrow y = -10$$

$\therefore$ The coordinates of A are $(3, -10)$

5.How can you identify the three points A, B and C are collinear?

If the sum of any two points distances (AB, BC, AC) equals the third (e.g., AB + BC = AC), then the three points A, B and C are collinear

6.Write the formulae to find:

i)The point which divides the joining of the two points $(x_1, y_1)$ and $(x_2, y_2)$ in the ratio $m_1 : m_2$ internally.

ii)The mid-point on the line joining the points $(x_1, y_1)$, and $(x_2, y_2)$

i)The point which divides the joining of the two points $(x_1, y_1)$, and $(x_2, y_2)$in the ratio $m_1 : m_2$ internally.

$$P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} , \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

ii)The mid-point on the line joining the points $(x_1, y_1)$ $and$ $(x_2, y_2)$

Midpoint formula $= \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$

## LEVEL-2: SHINING STAR

### 8 MARKS QUESTION ANSWERS

1.Q(0, 1) is equidistant from P (5, −3) and R $(x, 6)$ Find the value of 'x'. Determine the lengths of QR and PR

Solution:

Given, Q (0, 1) is equidistant from P (5, −3) and R $(x, 6)$

We know that the distance between the two points is given by the Distance Formula,

By the given condition, PQ = QR (using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ )

$$\sqrt{(0 - 5)^2 + (1 - (-3))^2} = \sqrt{(x - 0)^2 + (6 - 1)^2}$$

$$\Rightarrow \sqrt{(-5)^2 + (1 + 3))^2} = \sqrt{x^2 + 5^2}$$

$$\Rightarrow \sqrt{(-5)^2 + 4^2} = \sqrt{x^2 + 5^2}$$

$$\Rightarrow \sqrt{25 + 16} = \sqrt{x^2 + 25}$$

Squaring on both sides

$$\Rightarrow 25 + 16 = x^2 + 25$$

$$\Rightarrow x^2 = 16 \quad \Rightarrow \quad x = \pm\sqrt{16} = \pm 4$$

$$\therefore x = +4 \text{ or } -4$$

i) If $x = +4$

*then* $P(5, -3)$ $Q(0, 1)$ *and* $R(4, 6)$

$$QR = \sqrt{(4 - 0)^2 + (6 - 1)^2} = \sqrt{4^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41} \text{ units}$$

$$PR = \sqrt{(4 - 5)^2 + (6 - (-3))^2} = \sqrt{(1)^2 + (6 + 3))^2} = \sqrt{(1)^2 + (9))^2}\ \sqrt{1 + 81} = \sqrt{82} \text{ units}$$

ii) If $x = -4$

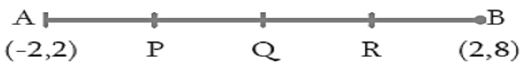
then $P(5, -3)$ $Q(0, 1)$ and $R(-4, 6)$

$$QR = \sqrt{(-4 - 0)^2 + (6 - 1)^2} = \sqrt{(-4)^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41} \text{ units}$$

$$PR = \sqrt{(-4 - 5)^2 + \left(6 - (-3)\right)^2} = \sqrt{(-9)^2 + (6 + 3))^2}$$

$$= \sqrt{(-9)^2 + (9))^2} = \sqrt{81 + 81} = \sqrt{2 \times 81} = 9\sqrt{2} \text{ units}$$

2.Find the coordinates of the points which divides the line segment joining A (−2, 2) and B(2, 8) into four equal parts.




Solution: From the Figure,

By observation, that points P, Q, R divides the line segment A (−2, 2) and B (2, 8) into four equal parts

using midpoint formula $= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Point Q divides the line segment AB into two equal parts

Hence, midpoint of AB is $Q = \left( \frac{-2+2}{2}, \frac{2+8}{2} \right) = \left( \frac{0}{2}, \frac{10}{2} \right) = (0, 5)$

Point P divides the line segment AQ into two equal parts

Hence, midpoint of AQ is $P = \left( \frac{-2+0}{2}, \frac{2+5}{2} \right) = \left( \frac{-2}{2}, \frac{7}{2} \right) = \left(-1, \frac{7}{2}\right)$

Point R divides the line segment BQ into two equal parts

Hence, midpoint of BQ is $R = \left( \frac{2+0}{2}, \frac{8+5}{2} \right) = \left( \frac{2}{2}, \frac{13}{2} \right) = \left(1, \frac{13}{2}\right)$

3. In what ratio does the point (−4, 6) divides the line segment joining the points A (−6, 10) and B (3, −8).

Solution: *Let* the point P (−4, 6) divides the line segment joining the points

A (−6, 10) and B (3, −8) be *in the ratio be* $k : 1$

Here $x_1 = -6$ , $y_1 = 10$ ,

$x_2 = 3$ , $y_2 = -8$

$m = k$ *and* $n = 1$


By using Section formula $P(x, y) = \left( \frac{mx_2 + nx_1}{m+n} , \frac{my_2 + ny_1}{m+n} \right)$

$P(-4, 6) = \left( \frac{k\times 3 + 1\times(-6)}{k+1} , \frac{k\times(-8) + 1\times 10}{k+1} \right) = \left( \frac{3k-6}{k+1} , \frac{-8k+10}{k+1} \right)$

So, $-4 = \frac{3k-6}{k+1} \Rightarrow -4(k+1) = 3k - 6$

$\Rightarrow -4k - 4 = 3k - 6 \Rightarrow -4k - 3k = 4 - 6 \Rightarrow -7k = -2 \Rightarrow 7k = 2 \Rightarrow k = \frac{2}{7}$

$\therefore$ the required ratio be $k : 1 = 2 : 7$

## 2 MARKS QUESTION ANSWERS

1.Find the coordinates of mid-point on the line segment joining the points (Cos 0°, 0) and (0, Sin 90°).

using midpoint formula $= \left( \frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$

mid-point on the line segment joining the points (Cos 0°, 0) and (0, Sin 90°)

$= \left( \frac{\cos 0° + 0}{2} , \frac{0 + \sin 90°}{2} \right) = \left( \frac{1+0}{2} , \frac{0+1}{2} \right) = \left( \frac{1}{2} , \frac{1}{2} \right)$

{since from trigonometric ratios Cos 0° = 1 and Sin 90° = 1 }

2.Find the ratio in which the line segment joining A(1, − 5) and B (−4, 5) is divided by the X-axis?

Solution: Let the required ratio be $k : 1$

Section formula $P(x, y) = \left( \frac{mx_2 + nx_1}{m+n} , \frac{my_2 + ny_1}{m+n} \right)$

Then by the section formula, the coordinates of the point which divides AB in the ratio $k : 1$ *are*

$\left( \frac{k\times(-4) + 1\times 1}{k+1} , \frac{k\times 5 + 1\times(-5)}{k+1} \right) = \left( \frac{-4k+1}{k+1} , \frac{5k-5}{k+1} \right)$

We know that y − coordinate of any point on $X$ axis is 0.

So $\frac{5k-5}{k+1} = 0 \Rightarrow 5k - 5 = 0 \Rightarrow 5k = 5 \Rightarrow k = 1$

$\therefore$ X axis divides it in the ratio 1: 1

3.Find the value of ' y 'for which the distance between the points P (2, − 3) and Q (10, y) is 10 units?

Solution: Given, Distance between points A (2, −3) and B (10, $y$) is 10 units. i.e. AB= 10 units

We know that the distance between the two points is given by the Distance Formula,

using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$AB = 10 \; units$$

$$\Rightarrow \sqrt{(10-2)^2 + \left(y-(-3)\right)^2} = 10 \quad \Rightarrow \sqrt{8^2 + (y+3)^2} = 10$$

$$\Rightarrow \sqrt{64 + (y+3)^2} = 10$$

Squaring on both sides

$$64 + (y+3)^2 = 100 \Rightarrow (y+3)^2 = 100 - 64$$

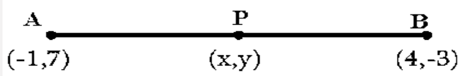
$$\Rightarrow (y+3)^2 = 36 \Rightarrow y+3 = \pm\sqrt{36} = \pm 6$$

$$y+3 = 6 \quad or \quad y+3 = -6 \quad \Rightarrow \quad y = 6-3 = 3 \quad or \quad y = -6-3 = -9$$

$$\therefore y = 3 \; or -9$$

4.Find the coordinates of the point which divides the line joining (− 1, 7) and (4, −3) in the Ratio 2: 3

Solution: Given, Let $P(x, y)$ be the required point.




Let A(−1, 7) and B(4, −3) m: n = 2:3

Hence $x_1 = -1$ , $y_1 = 7$ ,

$x_2 = 4$ , $y_2 = -3$

By Section formula $P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$$P(x,y) = \left(\frac{2\times4 + 3\times(-1)}{2+3}, \frac{2\times(-3) + 3\times7}{2+3}\right) = \left(\frac{8-3}{5}, \frac{-6+21}{5}\right) = \left(\frac{5}{5}, \frac{15}{5}\right) = (1, 3)$$

∴ The co-ordinates of point P are (1, 3).

5.Find the point on X-axis which is equidistant from A (2, −5) and B (−2, 9)?

Solution: Given, Since the point is on x-axis the co-ordinates are P $(x, 0)$.

We have to find a point on x-axis which is equidistant from A (2, −5) and B (−2, 9).

We know that the distance between the two points is given by the Distance Formula,

By the given condition, PA = PB So $PA^2 = PB^2$

i.e. $(x-2)^2 + 25 = (x+2)^2 + 81$ (using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ )

i.e. $x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

i.e. $\cancel{x^2} - 4x + \cancel{4} + 25 = \cancel{x^2} + 4x + \cancel{4} + 81$

i.e. $-4x + 25 = 4x + 81$

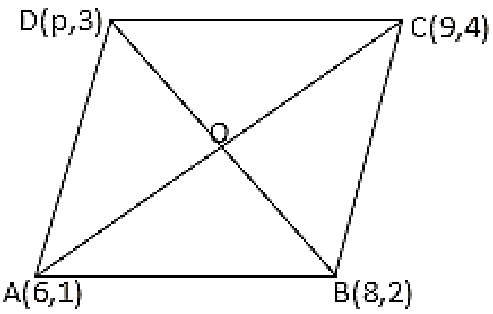
i.e. $-4x - 4x = 81 - 25$

i.e. $-8x = 56$

i.e. $x = \frac{56}{-8} = -7$

Therefore, the point equidistant from the given points on the x axis is P $(-7, 0)$.

6.Find the value of 'p', if the points A (6, 1), B(8, 2) C(9,4) and D(p, 3) are the vertices of a parallelogram.




Solution: We know that diagonals of a parallelogram bisect each other.

So, the coordinates of the mid-point of AC = coordinates of the mid-point of BD

i.e., $\left[\frac{6+9}{2}, \frac{1+4}{2}\right] = \left[\frac{8+p}{2}, \frac{2+3}{2}\right]$ $\{\because$ midpoint formula $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\}$

$$\Rightarrow \left[\frac{15}{2}, \frac{5}{2}\right] = \left[\frac{8+p}{2}, \frac{5}{2}\right]$$

$$\Rightarrow \frac{15}{2} = \frac{8+p}{2}$$

$$\Rightarrow 15 = 8 + p$$

$$\Rightarrow p = 15 - 8 = 7$$

$\therefore$ the value of $'p' = 7$

7.What are the coordinates of the point which lies on both the axes?

The coordinates of the point that lies on both the axes are (0, 0), known as the origin

8.Find the distance between the points (3,0) and (0,4)

distance between two points $formula = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

distance between two points (3,0) and (0,4) $= \sqrt{(0-3)^2 + (4-0)^2} = \sqrt{(-3)^2 + (4)^2}$

$$= \sqrt{(9 + 16} = \sqrt{25} = 5units$$

9.Write the section formula in internal division for the points $(x_1, y_1), (x_2, y_2)$the ratio $m_1 : m_2$

Section formula $P(x,y) = \left( \frac{m_1x_2+ m_2x_1}{m_1+m_2} , \frac{m_1y_2+ m_2y_1}{m_1+m_2} \right)$

10.What is the distance between the points (a, b) and (a, − b)?

distance between two points $formula = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

distance between two points (3,0) and (0,4) = $\sqrt{(a-a)^2 + (-b-b)^2}$

$$= \sqrt{0^2 + (-2b)^2}$$

$$= \sqrt{4b^2} = 2b \; units$$

11.The midpoint of on the line joining the points (− 4, 2) and (4, −2) is (0, 2). (True/False)

midpoint formula $= \left( \frac{x_1+ x_2}{2} , \frac{y_1+ y_2}{2} \right)$

midpoint of on the line joining the points (– 4, 2) and (4, −2)

$$= \left[ \frac{-4+4}{2} , \frac{2+(-2)}{2} \right] = \left[ \frac{0}{2} , \frac{2-2}{2} \right] = [\,0\,,\,0\,]$$

12.In which ratio that y-axis divide the line joining the points (5, 2) and (3, −2)

Solution: Let the required ratio be $k : 1$

Section formula $P(x,y) = \left( \frac{mx_2+ nx_1}{m+n} , \frac{my_2+ ny_1}{m+n} \right)$

Then by the section formula, the coordinates of the points the points (5, 2) and B (3, −2) which divides AB in the ratio $k : 1$ *are*

$$\left( \frac{k\times 3 + 1\times 5}{k+1} , \frac{k\times(-2)+ 1\times 2}{k+1} \right) = \left( \frac{3k+5}{k+1} , \frac{-2k+2}{k+1} \right)$$

We know that x − coordinate of any point on $y$ axis is 0.

So $\frac{3k+5}{k+1} = 0 \implies 3k + 5 = 0 \implies 3k = -5 \implies k = \frac{-5}{3}$

$\therefore$ $y$ axis divides it in the ratio − 5: 3

13.If the points P and Q are the trisecting points on the line segment AB, what ratio does P and Q divides the line AB?

Points P and Q trisecting line segment AB divide it into three equal parts (AP = PQ = QB);

P divides AB internally in the ratio 1:2, while Q divides AB internally in the ratio 2:1, relative to point A.

14.Find the distance between the points (a Cos $\theta$, 0) and (0, a Sin $\theta$)

distance between two points $formula = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

distance between two points (a Cos θ, 0) and (0, a Sin θ)

$$= \sqrt{(0 - a\cos\theta)^2 + (a\sin\theta - 0)^2}$$

$$= \sqrt{(- a\cos\theta)^2 + (a\sin\theta)^2}$$

$$= \sqrt{a^2\, cos^2\theta + a^2\, sin^2\theta}$$

$$= \sqrt{a^2\,(cos^2\theta + sin^2\theta)}$$

$$= \sqrt{a^2\,(1)}$$

$$= \sqrt{a^2}$$

$$= a \quad units$$

# 8.INTRODUCTION TO TRIGONOMETRY

**[7 Marks] [1 + 2 + 4 = 7 M]**

### LEVEL-1: RISING STAR

## 4 MARKS QUESTION ANSWERS

1.State any three trigonometric Identities.

$i)\ \sin^2\theta + \cos^2\theta = 1$ $\quad ii)\ \sec^2\theta - \tan^2\theta = 1$ $\quad iii)\ \mathrm{cosec}^2\theta - \cot^2\theta = 1$

2.Find the value of a) sin 60° cos 30°+ sin 30° cos 60° b)2 $\tan^2 45° + \cos^2 30° - \sin^2 60°$

a) sin 60° cos 30° + sin 30° cos 60°

$$\sin 60° \cos 30° + \sin 30° \cos 60° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

$$[\because \sin 60° = \frac{\sqrt{3}}{2},\ \cos 30° = \frac{\sqrt{3}}{2},\ \sin 30° = \frac{1}{2},\ \cos 60° = \frac{1}{2}]$$

b)2 $\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$2.\ \tan^2 45° + \cos^2 30° - \sin^2 60° = 2(1)^2 + (\frac{\sqrt{3}}{2})^2 - (\frac{\sqrt{3}}{2})^2 = 2 \times 1 + \frac{3}{4} - \frac{3}{4} = 2 + 0 = 2$$

$$[\because \sin 60° = \frac{\sqrt{3}}{2},\ \cos 30° = \frac{\sqrt{3}}{2},\ \tan 45° = 1]$$

3.Write other trigonometric ratios of ∠A in terms of sec A

Solution: We know that, trigonometric function $\cos A = \frac{1}{secA}$ -----(i)

Also $\sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A$

$$\Rightarrow \sin^2 A = 1 - (\frac{1}{\sec A})^2$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\Rightarrow \sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}}$$

$$\Rightarrow \sin A = = \frac{\sqrt{\sec^2 A - 1}}{\mathrm{Sec}\, A} \quad ------(ii)$$

Using trigonometric identity $\sec^2 A - \tan^2 A = 1 \Rightarrow \tan^2 A = \sec^2 A - 1$

trigonometric function $\tan A = \sqrt{\sec^2 A - 1}$ -----(iii)

trigonometric function $\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A - 1}}$-----(iv) [∵ substituing equation( iii) ]

trigonometric function $\mathrm{cosec}\, A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\mathrm{Sec}\, A}} = \frac{\mathrm{Sec}\, A}{\sqrt{\sec^2 A - 1}}$-----(v)

[∵ substituing equation( ii ) and simplifying]

4.Reproduce $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ (or) Show that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Solution: L.H.S = $\sqrt{\frac{1+\text{Sin}A}{1-\text{Sin }A}}$

On multiplying both numerator and denominator by $\sqrt{1+\text{ Sin}A}$ then

$$\text{L.H.S} = \sqrt{\frac{1+\text{Sin}A}{1-\text{Sin }A}} \times \sqrt{\frac{1+\text{Sin}A}{1+\text{Sin }A}}$$

$$= \sqrt{\frac{(1+\text{Sin}A)\ (1+\text{Sin}A)}{(1-\text{Sin }A)\ (1+\text{Sin}A)}}$$

$$= \sqrt{\frac{(1+\text{Sin}A)^2}{(1-\sin^2 A)}} \quad [\because (a+b)(a-b) = a^2 - b^2\ ]$$

$$= \sqrt{\frac{(1+\text{Sin}A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$= \sec A + \tan A$

$= \text{R.H.S}$

$\therefore \text{L.H.S} = \text{R.H.S}$

5.Reproduce $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$ as $7 + \tan^2 A + \cot^2 A$

Solution: $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2\sin A \,\text{cosec } A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

Rearranging and using $\sec A = \frac{1}{\cos A}$ and $\text{cosec } A = \frac{1}{\sin A}$ then

$= (\sin^2 A + \cos^2 A) + (\text{cosec}^2 A + \sec^2 A) + 2\sin A \times \frac{1}{\sin A} + 2\cos A \times \frac{1}{\cos A}$

$= 1 + (1+\cot^2 A) + (1+\tan^2 A) + 2 + 2$

| |
|---|
| $\sin^2 A + \cos^2 A = 1$ |
| $\sec^2 A - \tan^2 A = 1 \Rightarrow \sec^2 A = 1 + \tan^2 A$ |
| $\text{cosec}^2 A - \cot^2 A = 1 \Rightarrow \text{cosec}^2 A = 1 + \cot^2 A$ |

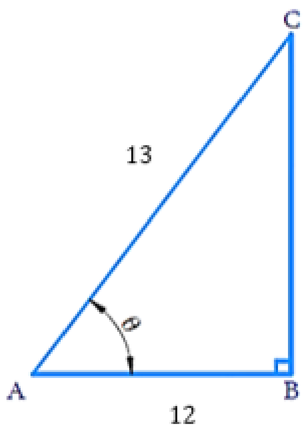
$= 7 + \cot^2 A + \tan^2 A$

$\therefore (\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

## 2 MARKS QUESTION ANSWERS

1. If $\sec\theta = \frac{13}{12}$ determine all trigonometric ratios.

Solution: Let $\Delta ABC$ be a right-angled triangle, right angled at point B




Given $\sec\theta = \frac{13}{12}$ ,

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side to } \angle\theta} = \frac{AC}{AB}$ i.e., $\sec\theta = \frac{13}{12} = \frac{AC}{AB}$

using Pythagoras theorem

$AC^2 = AB^2 + BC^2 \Rightarrow 13^2 = 12^2 + BC^2 \Rightarrow 13^2 - 12^2 = BC^2$

$\Rightarrow 169 - 144 = BC^2 \Rightarrow 25 = BC^2 \Rightarrow BC = \pm\sqrt{25} \Rightarrow BC = \pm 5$

Consider positive BC = 5

$\sin\theta = \dfrac{\text{opposite side to } \angle\theta}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{5}{13}$

$\cos\theta = \dfrac{\text{adjacent side to } \angle\theta}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{12}{13}$

$\tan\theta = \dfrac{\text{opposite side to } \angle\theta}{\text{adjacent side to } \angle\theta} = \dfrac{BC}{AB} = \dfrac{5}{12}$

$\text{cosec}\theta = \dfrac{\text{hypotenuse}}{\text{opposite side to } \angle\theta} = \dfrac{AC}{BC} = \dfrac{13}{5}$

$\cot\theta = \dfrac{\text{adjacent side to } \angle\theta}{\text{opposite side to } \angle\theta} = \dfrac{AB}{BC} = \dfrac{12}{5}$

## Alternate method

Solution: Given $\sec\theta = \frac{13}{12} \Rightarrow \cos\theta = \frac{12}{13}$ $\left(\because \sec A = \frac{1}{\cos A}\right)$

$\sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A = 1 - \left(\frac{12}{13}\right)^2 = 1 - \frac{144}{169} = \frac{25}{169}$

$\text{Sin } A = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$ consider positive so $\text{Sin } A = \frac{5}{13}$

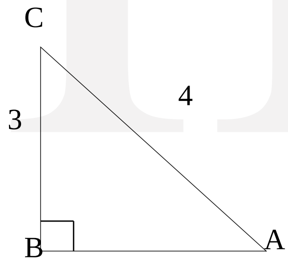
$\text{cosec } A = \frac{13}{5}$ $\left(\because \text{cosec } A = \frac{1}{\sin A}\right)$

$\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\frac{5}{13}}{\frac{12}{13}} = \dfrac{5}{12}$ $\quad \therefore \tan A = \dfrac{5}{12}$

$\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\frac{5}{12}} = \dfrac{12}{5}$ $\quad \therefore \cot A = \dfrac{12}{5}$

2. If $\sin A = \frac{3}{4}$ determine cos A, tan A.

Solution: Given $\sin A = \frac{3}{4}$




$\sin A = \dfrac{\text{opposite side to} \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC}$, $AC = 4$, $BC = 3$

using Pythagoras theorem $AC^2 = AB^2 + BC^2$

$4^2 = AB^2 + 3^2 \Rightarrow 16 = AB^2 + 9 \Rightarrow AB^2 = 16 - 9 \Rightarrow AB^2 = 7$

$\Rightarrow AB = \pm\sqrt{7}$ taken positive then $AB = \sqrt{7}$

$\cos A = \dfrac{\text{adjacent side to} \angle A}{\text{hypotenuse}} = \dfrac{BC}{AC}$, $\cos A = \dfrac{\sqrt{7}}{4}$

$\tan A = \dfrac{\text{opposite side to} \angle A}{\text{adjacent side to} \angle A} = \dfrac{BC}{AB}$, $\tan A = \dfrac{3}{\sqrt{7}}$

## Alternate method

Solution: Given $\sin A = \frac{3}{4}$

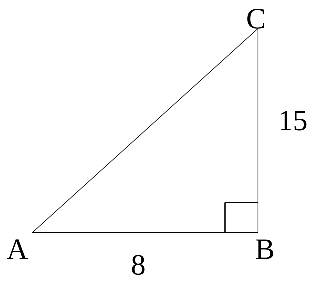
Using trigonometric identity $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \cos^2 A = 1 - \sin^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$

$\cos A = \pm\sqrt{\frac{7}{16}} = \pm\frac{\sqrt{7}}{4}$ consider positive, $\cos A = \frac{\sqrt{7}}{4}$

$\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \dfrac{3}{\sqrt{7}}$ $\quad \therefore \tan A = \dfrac{3}{\sqrt{7}}$

3. If 15 cot A = 8 determine sin A, sec A




Solution: Let us consider a right-angled ΔABC, right angled at B.

Given 15 cot A $= 8 \Longrightarrow$ cot A $= \frac{8}{15}$

$\cot A = \frac{\text{adjacent side to } \angle A}{\text{opposite side to } \angle A} = \frac{AB}{BC} = \frac{8}{15}$

using Pythagoras theorem $AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 15^2 \Longrightarrow AC^2 = 64 + 225 \Longrightarrow AC^2 = 289 \Longrightarrow AC = \pm\sqrt{289} \Longrightarrow AC = \pm 17$

Consider positive $AC = 17$

$\sin A = \frac{\text{opposite side to} \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{15}{17}$

$\sec A = \frac{\text{hypotenuse}}{\text{adjacent side to } \angle A} = \frac{AC}{AB} = \frac{17}{8}$

## Alternate method

Solution: Given 15 cot A $= 8 \Longrightarrow$ cot A $= \frac{8}{15}$

Using trigonometric identity $\text{cosec}^2 A - \cot^2 A = 1 \Rightarrow \text{cosec}^2 A = 1 + \cot^2 A$

$\Longrightarrow \text{cosec } A = \pm\sqrt{1 + \cot^2 A}$ consider positive $\text{cosec } A = \sqrt{1 + \cot^2 A}$

$\text{cosec } A = \sqrt{1 + \cot^2 A} = \sqrt{1 + (\frac{8}{15})^2} = \sqrt{1 + \frac{64}{225}} = \sqrt{\frac{289}{225}} = \frac{17}{15}$

$\text{cosec } A = \frac{1}{\sin A}$ *so* $\sin A = \frac{15}{17}$

$\cot A = \frac{\cos A}{\sin A} \Longrightarrow \frac{8}{15} = \cos A = \cot A \times \sin A \Longrightarrow \cos A = \frac{8}{15} \times \frac{15}{17} = \frac{8}{17}$

$\sec A = \frac{1}{\cos A} = \frac{1}{\frac{8}{17}} = \frac{17}{8}$ $\quad \therefore \sec A = \frac{17}{8}$

4. If $\cot\theta = \frac{7}{8}$ determine $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

Solution: Given $\cot\theta = \frac{7}{8}$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$ $\quad [\because (a+b)(a-b) = a^2 - b^2]$

$= \frac{\cos^2\theta}{\sin^2\theta}$

| |
|---|
| $\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta$ |
| $\sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta$ |

$= \cot^2\theta$ $\quad [\because \cot\theta = \frac{\cos\theta}{\sin\theta}]$

$= (\frac{7}{8})^2 = \frac{49}{64}$

$\therefore \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{49}{64}$

5. If $\sin(A-B)=\frac{1}{2}$ , $\cos(A+B)=\frac{1}{2}$ measure the values of A, B

Solution : Since, $\sin(A-B)=\frac{1}{2}=\sin 30°$

therefore, $A-B=30°$ ......( i )

Also, $\cos(A+B)=\frac{1}{2}=\cos 60°$

therefore, $A+B=60°$ .....(ii )

Solving (i) and (ii), we get : $A=45°$ and $B=15°$

$$\begin{array}{r} A-B=30° \\ + \quad A+B=60° \\ \hline 2A=90° \Rightarrow A=45° \end{array}$$

sub A 45° in eq (ii ) $\Rightarrow 45°+B=60° \Rightarrow B=60°-45°=15°$

6. If $\tan(A+B)=\sqrt{3}$, $\tan(A-B)=\frac{1}{\sqrt{3}}$ measure the values of A, B

Given that $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$

$\tan(A+B)=\tan 60°$ ($\because \tan 60°=\sqrt{3}$ )

$\Rightarrow A+B=60°$---------- (i )

$\tan(A-B)=\frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B)=\tan 30°$ ($\because \tan 30°=\frac{1}{\sqrt{3}}$ )

$\Rightarrow A-B=30°$---------- (ii )

On adding both equations (i) and (ii) we get

$A+B+A-B=60°+30° \Rightarrow 2A=90° \Rightarrow A=45°$

Substitute A= 45° in eq (i) we get

$A+B=60° \Rightarrow B=60°-A \Rightarrow B=60°-45° \Rightarrow B=15°$

$\therefore \angle A=45°, \angle B=15° \quad (A>B)$

## LEVEL-2: SHINING STAR

### 4 MARKS QUESTION ANSWERS

1. Find the values of

a) $\frac{\sin 90° \tan 45° + \cos 0° \sec 60°}{\sec^2 45° + \mathrm{cosec}^2 45°}$

$$\frac{\sin 90° \tan 45° + \cos 0° \sec 60°}{\sec^2 45° + \mathrm{cosec}^2 45°} = \frac{1\times 1 + 1\times 2}{\sqrt{2}^2 + \sqrt{2}^2} = \frac{1+2}{2+2} = \frac{3}{4}$$

{ $\because$ sin 90° =1, tan45°=1, cos0°=1 sec60°= 2, sec45°= $\sqrt{2}$, cosec45°=$\sqrt{2}$ }

b) $\sqrt{2}\sin 45° + \sqrt{3}\cos 30° + \mathrm{cosec}\, 90°$

$$\sqrt{2}\sin 45° + \sqrt{3}\cos 30° + \mathrm{cosec}\, 90° = \sqrt{2}\times\frac{1}{\sqrt{2}} + \sqrt{3}\times\frac{\sqrt{3}}{2} + 1 = 1 + \frac{3}{2} + 1 = 2 + \frac{3}{2} = \frac{7}{2}$$

{ $\because \sin 45° = \frac{1}{\sqrt{2}}$ , $\cos 30° = \frac{\sqrt{3}}{2}$ , cosec 90°=1 }

2. Write cos A, tan A, sec A in terms of sin A

Solution: $\sin^2 A + \cos^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A \Rightarrow \cos A = \sqrt{1-\sin^2 A}$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1-\sin^2 A}}$$

3. Reproduce $(\operatorname{cosec}\theta - \cot\theta)^2$ as $\frac{(1-\cos\theta)}{(1+\cos\theta)}$ (or) Show that $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{(1-\cos\theta)}{(1+\cos\theta)}$

Solution:

$$\text{L.H.S} = (\operatorname{cosec}\theta - \cot\theta)^2$$
$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \quad \left\{ \because \operatorname{cosec}\theta = \frac{1}{\sin\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta} \right\}$$
$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$
$$= \frac{(1+\cos\theta)\,(1-\cos\theta)}{1-\cos^2\theta} \quad (\because 1-\cos^2\theta = \sin^2\theta)$$
$$= \frac{(1-\cos\theta)\,(1-\cos\theta)}{(1+\cos\theta)\,(1-\cos\theta)} = \frac{(1-\cos\theta)}{(1+\cos\theta)} = \text{RHS}$$

4. Reproduce $\frac{\cot A - \cos A}{\cot A + \cos A}$ as $\frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Solution: $\text{L.H.S} = \frac{\cot A - \cos A}{\cot A + \cos A}$

$$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A} \quad \left\{ \because, \cot A = \frac{\cos A}{\sin A} \right\}$$
$$= \frac{\cos A\left(\frac{1}{\sin A} - 1\right)}{\cos A\left(\frac{1}{\sin A} + 1\right)}$$
$$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \quad \left(\because \operatorname{cosec} A = \frac{1}{\sin A}\right)$$
$$= \text{RHS}$$

## 2 MARK QUESTION ANSWERS

1.sec A (1 – sin A) (sec A + tan A) = 1 Justify?

Solution: L.H.S = sec A (1 – sin A) (sec A + tan A

$$= \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right) \quad \left(\because \sec A = \frac{1}{\cos A}, \tan A = \frac{\sin A}{\cos A}\right)$$
$$= \frac{(1-\sin A)\,(1+\sin A)}{\cos^2 A}$$
$$= \frac{(1-\sin^2 A)}{\cos^2 A} \quad (\because a^2 - b^2 = (a+b)\,(a-b))$$
$$= \frac{\cos^2 A}{\cos^2 A} \quad (\because 1-\sin^2 A = \cos^2 A)$$
$$= 1$$
$$= \text{RHS}$$

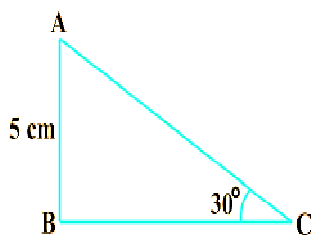
2.In a right-angled triangle ΔABC, ∠B = 90°, AB = 5 cm, ∠ACB = 30°. Determine the lengths BC and AC.

Solution: To find the length of the side BC, we will choose the trigonometric ratio involving BC and the given side AB. Since BC is the side adjacent to angle C and AB is the side opposite to angle C,

therefore $\tan C = \frac{\text{opposite side to } \angle C}{\text{adjacent side to } \angle C} = \frac{AB}{BC}$

$\Rightarrow \tan 30^\circ = \frac{5}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{BC}$ $\left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$

$\Rightarrow BC = 5\sqrt{3}$ cm



To find the length of the side AC, we consider

$$\sin C = \frac{\text{opposite side to } \angle C}{\text{hypotenuse}} = \frac{AB}{AC} \implies \sin 30° = \frac{5}{AC}$$

$$\implies \frac{1}{2} = \frac{5}{AC} \quad (\because \sin 30° = \frac{1}{2})$$

$$\implies AC = 10 \text{ cm}$$

3. In a $\Delta ABC$, $\angle B = 90°$, If $\tan A = \frac{1}{\sqrt{3}}$ find the value of $\sin A \cos C + \cos A . \sin C$

In a $\Delta ABC$, $\angle B = 90°$, $\tan A = \frac{1}{\sqrt{3}} = \tan 30°$ i.e $A = 30°$ and $C = 60°$ (using angle sumproperty)

Now put $A = 30°$ and $C = 60°$ in $\sin A \cos C + \cos A . \sin C$ then

$\sin A \cos C + \cos A . \sin C = \sin 30° . \cos 60° + \cos 30° . \sin 60°$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

[ $\because \sin 60° = \frac{\sqrt{3}}{2}$ , $\cos 30° = \frac{\sqrt{3}}{2}$ , $\sin 30° = \frac{1}{2}$ , $\cos 60° = \frac{1}{2}$ ]

## 1 MARK QUESTION ANSWERS

1. Which trigonometric ratio equal to Adj side/Hypotenuse? (B)

A) $\sin \theta$ B) $\cos \theta$ C) $\tan \theta$ D) $\cot \theta$

2.If a right-angled triangle. If $\tan A = 1$, then $\angle A =$ ( C )

A)0° B) 30° C) 45° D) 60°

$\tan A = 1 = \tan 45° \implies A = 45°$

3. If $\sin \theta = \cos \theta$ then $\tan \theta$ A )0 B) $\frac{1}{\sqrt{3}}$ C) $\sqrt{3}$ D) 1 ( D)

$\sin \theta = \cos \theta$ , $\sin 45° = \frac{1}{\sqrt{2}}$ , $\cos 45° = \frac{1}{\sqrt{2}}$ therefore $\theta = 45°$ $\tan \theta = \tan 45° = 1$

4. If $\tan A = 1$, then $2\sin A \cos A =$

A )0 B) $\frac{1}{2}$ C)1 D) 2 ( C )

$\tan A = 1 = \tan 45° \implies A = 45°$

$2\sin A \cos A = 2 \sin 45° \cos 45° = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2 \times \frac{1}{2} = 1$

5. $9 \sec^2 A - 9 \tan^2 A =$ .......... A )0 B) 1 C) 8 D) 9 ( D)

$9 \sec^2 A - 9 \tan^2 A = 9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9$

6. $\operatorname{cosec} \theta - \cot \theta = x$ then $\operatorname{cosec} \theta + \cot \theta =$ A) $-x$ B) $x + 1$ C ) $\frac{1}{x}$ D) $-\frac{1}{x}$ ( C )

$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$

$\implies \operatorname{cosec}^2 \theta - \cot^2 \theta = (\operatorname{cosec} \theta + \cot \theta)(\operatorname{cosec} \theta - \cot \theta) \implies 1 = (\operatorname{cosec} \theta + \cot \theta)(x)$

$$\implies \operatorname{cosec} \theta + \cot \theta = \frac{1}{x}$$

7. If 'θ' is an acute angle, as value of 'θ' Increase then value of cos θ

A) No change B) Increases C) Decreases D) Can't say ( C )

8. $\frac{2\tan 30°}{1+\tan^2 30°}$ = A) sin 60° B) cos 60° C) tan 60° D) sin 30° ( A )

$\frac{2tan\theta}{1+\tan^2\theta} = \sin 2\theta$ therefore $\frac{2tan30°}{1+\tan^2 30°} = \sin(2\times 30°) = \sin 60°$

9. $\frac{1+\tan^2 A}{1+\cot^2 A}$ = A) $\sec^2 A$ B) -1 C) $\cot^2 A$ D) $\tan^2 A$ ( D )

$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

10. If cot θ is undefined then θ = 0°

11. If $\sin\theta = \frac{a}{b}$ then $\tan\theta =$ A) $\frac{b}{a}$ B) $\frac{a}{\sqrt{b^2-a^2}}$ C) $\frac{a^2}{b^2}$ D) $\frac{b}{\sqrt{b^2-a^2}}$ ( B )

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{\frac{b^2-a^2}{b^2}} = \frac{\sqrt{b^2-a^2}}{b}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{a}{b}}{\frac{\sqrt{b^2-a^2}}{b}} = \frac{a}{\sqrt{b^2-a^2}}$

12. If $x = \sin\theta + \cos\theta$, $y = \sin\theta . \cos\theta$ which of the following is true?

$A)\ xy = 1$ $B)\ x^2 = 1 + 2y$ $C)\ xy = -1$ $D)\ x^2 = 1 - 2y$ ( B )

$x = \sin\theta + \cos\theta$, squaring on both sides

$x^2 = (\sin\theta + \cos\theta)^2$

$\Rightarrow x^2 = \sin^2\theta + 2\sin\theta\ \cos\theta + \cos^2\theta = \sin^2\theta + \cos^2\theta + 2\sin\theta\ \cos\theta = 1 + 2y$

13. If θ is acute angle then sin θ × cos θ × tan θ × cot θ × sec θ × cosec θ = .1.........

sin θ × cos θ × tan θ × cot θ × sec θ × cosec θ Rearranging then

(sin θ × cosec θ) × (cos θ × sec θ) ×( tan θ × cot θ)=1×1 ×1 =1

14. $\cos\theta \times \sqrt{1+\tan^2\theta} = \cos\theta \times \sec\theta = 1$

15. ABC is an Isosceles right-angled triangle ∠B = 90°, then $\tan^2 A - \cot^2 C = 0$

$\tan^2 45° - \cot^2 45°$. = 1-1 = 0

{∵ Isosceles right – angled triangle ∠B = 90°, so other two equal angles ∠A = 45° ∠C = 45° }

# 9.SOME APLICATIONS OF TRIGONOMETRY

**[11 Marks] [1 + 2 + 8 = 11 M]**

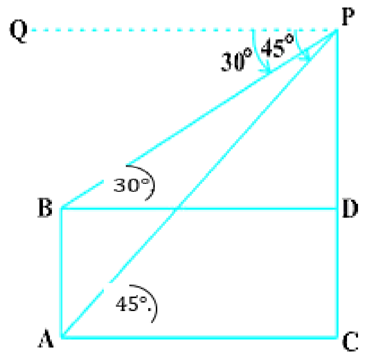
## LEVEL-1 : RISING STAR

### 8 MARKS QUESTION ANSWERS

1.From the top of a hill, a person observed the top and bottom of a 40m tall building with the angles of depression 30°and 45° respectively. Find the height of the hill and the distance between hill and the building

From the adjacent Fig. PC denotes the hill and AB denotes the 40 m tall building.

The height of the hill PC = H meters and




the distance between the two buildings, i. e., AC = D meters.

Observe that PB is a transversal to the parallel lines PQ and BD.

Therefore, ∠QPB and ∠PBD are alternate angles, and so are equal.

So $\angle PBD = 30°$. Similarly, $\angle PAC = 45°$. In right $\Delta PBD$, we have

$\frac{PD}{BD} = \tan 30° \implies \frac{PD}{BD} = \frac{1}{\sqrt{3}} \implies BD = PD\sqrt{3}$ ……(i)

In right $\Delta PAC$, we have

$\frac{PC}{AC} = \tan 45° \implies \frac{PC}{AC} = 1 \implies PC = AC$ ……(ii)

PC= PD + DC, therefore, PD + DC = AC

Since, $AC = BD$ and $DC = AB = 40$ m, we get $PD + 40 = BD = PD\sqrt{3}$

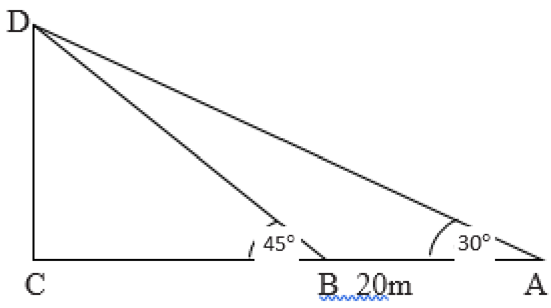
$\implies PD\sqrt{3} - PD = 40 \implies PD\left(\sqrt{3} - 1\right) = 40 \implies PD = \frac{40}{(\sqrt{3}-1)}$

$$PD = \frac{40}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{40(\sqrt{3}+1)}{\sqrt{3}^2 - 1^2} = \frac{40(\sqrt{3}+1)}{3-1} = \frac{40(\sqrt{3}+1)}{2} = 20\left(\sqrt{3}+1\right)m$$

So, the height of the hill $= \{20\left(\sqrt{3}+1\right) + 40\}m = 20\sqrt{3} + 20 + 40 = 20\sqrt{3} + 60 = 20(\sqrt{3}+3)m$

and the distance between the two buildings is also $20(\sqrt{3}+3)m$

2.The angle of elevation of the top of a tower from a point on the ground is 30° On moving 20m nearer to the tower, the angle of elevation increases to 45°. Find the height of the tower and the distance of the first point from the tower.'




Solution:

Let h be the height of the tower and the angle of elevation of the top of a tower from a point on the ground is 45° and on moving with distance 20m towards foot of the tower on the point B is 30°

Let AB= 20 m and BC = $x$ $m$

Now we have to find the height of the tower and distance of tower from first point A.

So, we use trigonometric ratios

From the Diagram

*In right $\Delta DBC$, we have*

Tan 45° $= \frac{DC}{BC} \implies 1 = \frac{h}{x} \implies x = h$ ...(i)

In right $\Delta DAC$, we have

$$\tan 30° = \frac{DC}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AB+BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x} \Rightarrow \Rightarrow h\sqrt{3} = 20 + x \quad \ldots..(ii)$$

$$h\sqrt{3} = 20 + h \quad (\because from\ (i))$$

$$\Rightarrow h\sqrt{3} - h = 20 \Rightarrow h(\sqrt{3} - 1) = 20 \Rightarrow h = \frac{20}{\sqrt{3}-1}$$

$$h = \frac{20}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{20(\sqrt{3}+1)}{\sqrt{3}^2 - 1^2} = \frac{20(\sqrt{3}+1)}{3-1} = \frac{20(\sqrt{3}+1)}{2} = 10(\sqrt{3}+1)m$$

the height of the tower = $10(\sqrt{3}+1)m$

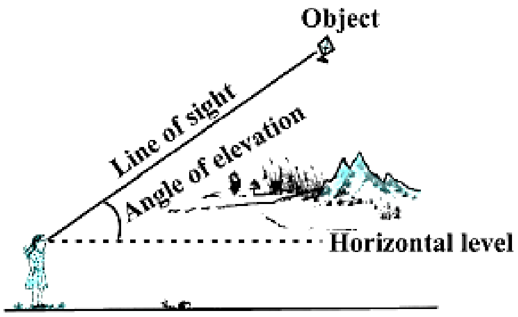
the distance of the first point from the tower AC= AB + BC

AC = $20 + 10(\sqrt{3}+1) = 20 + 10\sqrt{3} + 10 = 30 + 10\sqrt{3} = 10(3+\sqrt{3})m$

the distance of the first point from the tower = $10(3+\sqrt{3})m$

**2 MARKS QUESTION ANSWERS**

1.Define the angle of elevation with a simple rough diagram.




The angle of elevation of an object viewed, is the angle formed by the line of sight with the horizontal

when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

2.The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 30°. Find the height of the tower.

Solution: Let us consider the height of the tower as AB, distance between the foot of tower to the point on ground as BC.

In right ΔABC,

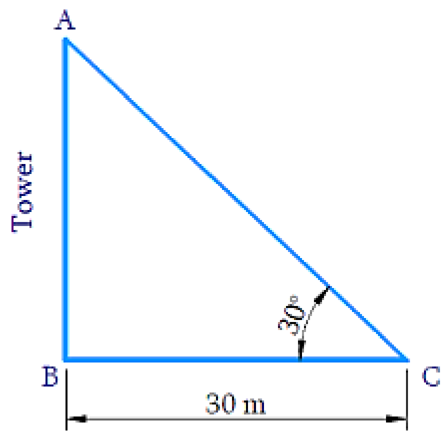
$$\tan C = \frac{\text{opposite side to } \angle C}{\text{adjacent side to } \angle C} = \frac{AB}{BC}$$

$$\Rightarrow \tan 30° = \frac{AB}{30}$$

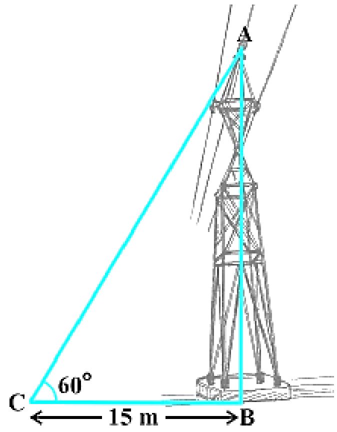
$$\Rightarrow AB = \frac{30}{\sqrt{3}} \text{ cm} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30} \quad (\because \tan 30° = \frac{1}{\sqrt{3}}) \Rightarrow$$

$$AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\ cm$$

$\therefore$ height of the tower = $10\sqrt{3}\ cm$



3.A tower standing vertically on the ground. From a point on the ground, which is 15m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60° Find the height of the tower.




Solution: From the fig Let AB represents the tower, CB is the distance of the point from the tower ∠ACB is the angle of elevation.

To solve the problem, we choose the trigonometric ratios

In right $\Delta ACB$, we have $\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{AB}{15} \Rightarrow \frac{1}{\sqrt{3}} \Rightarrow AB = 15\sqrt{3}\,m$

Hence, the height of the tower $= 15\sqrt{3}\,m$

4.Name any two real life situations where trigonometry used.

Trigonometry is used in construction for designing sloped roofs and measuring heights,

and in navigation (GPS, aviation, sailing)to determine positions and directions helping calculate distances and bearings for safe travel.

It also helps in fields like astronomy, game development, and even forensics to analyze trajectories.

## 1 MARK QUESTION ANSWERS

1. Define angle of Depression.

   The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object

2.Assertion (A): when the Sun's altitude is 60°, The ratio of the length of shadow of a vertical tower to its height is 1: $\sqrt{3}$

Reason (R): $\tan 60° = \sqrt{3}$

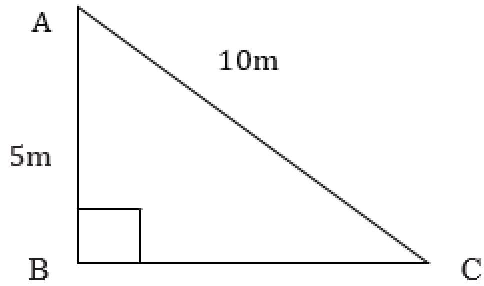
A) Both Assertion(A) and Reason (R) are true and the reason (R) is the correct explanation of assertion (A)✓

B) Both Assertion (A) and Reason (R) are true but the reason R is not the correct explanation of assertion (A)

C)Assertion (A) is true and Reason (R) is false. D) Assertion (A) is false but Reason (R) is true.

3.Outline the diagram to the below situation:

"A ladder of 10 m length touches a wall at a height 5m."




The situation forms a right-angled triangle

The vertical line represents the wall, with the point of contact marked at a height of 5 m

from the base. (AB= 5m)

The horizontal line represents the ground, meeting the wall at a 90-degree angle.

The hypotenuse (slanted line) represents the ladder, with a length of 10 m. (AC= 10m)

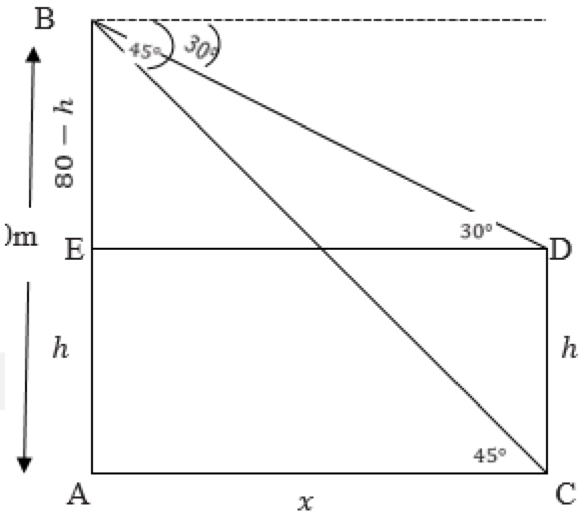
## LEVEL-2: SHINING STAR

### 8 MARKS QUESTIONS ANSWERS

1. A building and a tower are standing on the same level ground. From the top of the tower, the angle of depression of the top and bottom of the building are 30° and 45° respectively. If the height of the tower is 80m, find:

   (a) The height of the building.

   (b) The horizontal distance between the tower and the building.




Solution: Given the height of the tower = 80m and from the top of the tower,

the angle of depression of the top and bottom of the building are 30°and 45°respectively

let height of the building CD = h let the horizontal distance between the tower and the building AC = x

From the Diagram , In right ΔABC, we have

$\tan 45° = \frac{AB}{AC} \implies 1 = \frac{80}{x} \implies x = 80m$ ... (i)

the horizontal distance between the tower and the building AC = 80m

In right ΔEBD, we have

$tan30° = \frac{BE}{DE} \implies \frac{1}{\sqrt{3}} = \frac{AB-AE}{AC} = \frac{AB-CD}{AC}$ ($\because$ *from fig ACDE is a rectangle AC = 80m*)

$\implies \frac{1}{\sqrt{3}} = \frac{80-AE}{80}$ ($\because$ *from (i) AC = 80m = AB*)

$\implies \frac{1}{\sqrt{3}} = \frac{80-h}{80} \implies \frac{1}{\sqrt{3}} = 1 - \frac{h}{80} \implies \frac{h}{80} = 1 - \frac{1}{\sqrt{3}} \implies \frac{h}{80} = \frac{\sqrt{3}-1}{\sqrt{3}} \implies h = \frac{80(\sqrt{3}-1)}{\sqrt{3}}$

height of the building CD (h) $= \frac{80(\sqrt{3}-1)}{\sqrt{3}}\, m$

2.The angles of elevation of the top a tower from two points A and B on the same straight line from the base of the tower are complementary. If the distance between A and B is 50m, find the height of the tower and the distance of A and B from the base.

Solution:

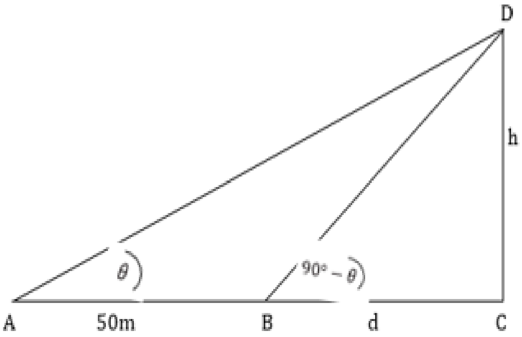
Let the height of tower CD = h

Given that: The angles of elevation of the top a tower from two points A and B on the same straight line from the base of the tower are complementary

So, let the angle of elevation of top of the tower are $\angle A = \theta$ *and* $\angle B = 90° - \theta$ and let BC= d distance between A and B, AB = 50m

Here, we have to find the height of tower.

From the Diagram , In right $\Delta ADC$, we have




$tanA = \frac{CD}{AC} \Longrightarrow tan\theta = \frac{h}{AB+BC} = \frac{h}{50+d}$

$\Longrightarrow tan\theta = \frac{h}{50+d} \quad \ldots\ldots..(i)$

From the Diagram , In right ΔBCD, we have

$tanB = \frac{CD}{BC} \Longrightarrow \tan(90° - \theta) = \frac{h}{d} \Longrightarrow \cot\theta = \frac{h}{d} \quad \ldots\ldots(ii)$

product of (i) and ( ii )

$tan\theta \times \cot\theta = \frac{h}{50+d} \times \frac{h}{d} \Longrightarrow 1 = \frac{h^2}{d(50+d)} \quad (\because tan\theta \times \cot\theta = 1)$

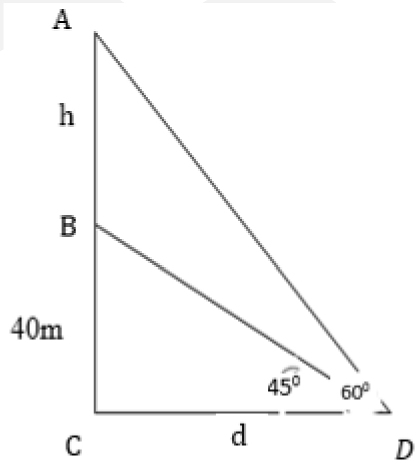
$\Longrightarrow h^2 = d(50+d) \Longrightarrow h = \pm\sqrt{d(50+d)}$

Height should be positive so we consider positive value $h = \sqrt{d(50+d)}\ m$

The distance between A and B is given as 50m. Assuming A and B are on the same side of the tower, The individual distances cannot be determined without another given value ($e.g.$, $one\ of\ the\ angles\ or\ one\ of\ the\ distances\ from\ the\ base$).

3.From a point on the ground the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 40 m high building are 45° and 60° respectively. Calculate the height of the tower.

Solution:




Let the height of the building BC = 40m
Let the height of the transmission tower AB=h meters
The total height from the ground to the top of the tower AC = h + 40
Let *D* be the point on the ground and CD=d m be the horizontal distance from the point to the base of the building.
In the right-angled Δ BCD

$tan45° = \frac{BC}{CD} \Longrightarrow 1 = \frac{40}{d} \Longrightarrow d = 40m$

In the right-angled Δ ACD

$tan60° = \frac{AC}{CD} \Longrightarrow \sqrt{3} = \frac{h+40}{d} \Longrightarrow \sqrt{3} = \frac{h+40}{40} \Longrightarrow 40\sqrt{3} = h + 40$

$\Longrightarrow h = 40\sqrt{3} - 40 = 40(\sqrt{3} - 1)\ m$

$h = 40(\sqrt{3} - 1)\ m$

Using the approximate value of $\sqrt{3} = 1.732$

we get h $\approx 40(1.732 - 1)m = 40 \times 0.732 = 29.28$m approximately

the height of the tower ( $AB$ )h = 29.28m

## 2 MARKS QUESTION ANSWERS

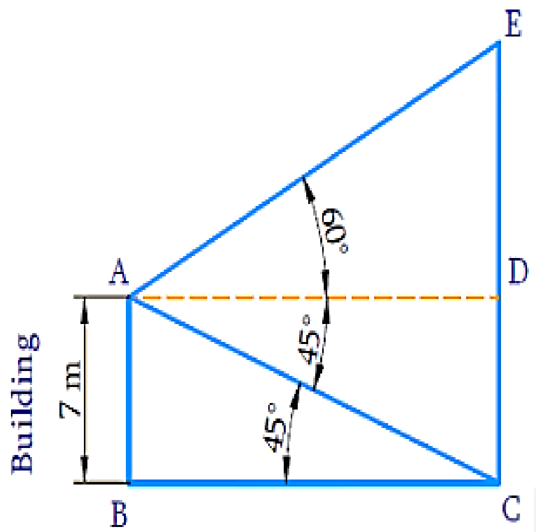
1. From the top of 7m high building, the angle of elevation of the top of the cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution: Let the height of the tower is CE and the height of the building is AB. The angle of elevation of the top E of the tower from the top A of the building is 60° and the angle of depression of the bottom C of the tower from the top A of the building is 45°.

Trigonometric ratio involving building height, tower height, angles and distances between them is tan θ

Draw AD||BC. Then, $\angle DAC = \angle ACB = 45°$ (alternate interior angles.)

$\tan 45° = \frac{AB}{BC} \Longrightarrow 1 = \frac{7}{BC} \Longrightarrow BC = 7$




ABCD is a rectangle, therefore BC=AD and AB= CD= 7

In In $\Delta ADE$

$\tan 60 = \frac{ED}{AD} \Longrightarrow \sqrt{3} = \frac{ED}{7} \Longrightarrow ED = 7\sqrt{3}$

height of the tower CE = ED + CD

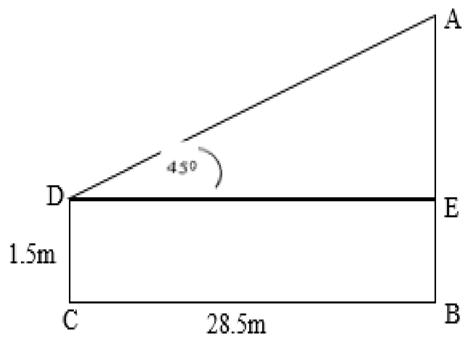
$= 7\sqrt{3} + 7$

$= 7(\sqrt{3} + 1)$

∴height of the tower = $= 7(\sqrt{3} + 1)$

2.The observer 1.5m tall is 28.5m away from a tower. The angle of elevation of the top of the tower from his eyes is 45°. What is the height of the tower?

Solution: Here, AB is the tower, CD the observer and ∠ADE the angle of elevation from the figure




In this case, ADE is a triangle, right-angled at E and we are required to find the height of the tower.

We have AB = AE + BE = AE + 1.5m and DE = CB = 28.5 m

To determine AE, we choose a trigonometric ratio, which involves both AE and DE. From the Diagram , In right $\Delta ADE$, we have

$tan45° = \frac{AE}{DE} \Longrightarrow 1 = \frac{AE}{DE} \Longrightarrow AE = DE$

$AE = BC = 28.5m$ (∵ from fig BCDE is a rectangle $BC = DE = 28.5m$)

Therefore, AE = 28.5 So the height of the tower (AB) = (28.5 + 1.5) m = 30 m.

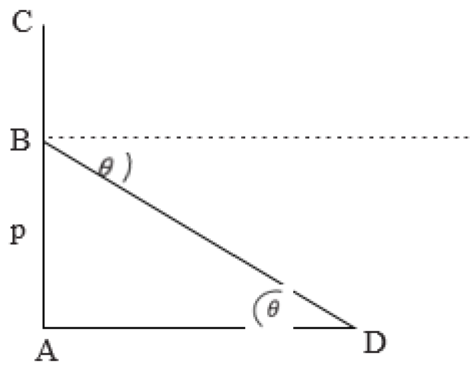
## 1 MARK QUESTION ANSWERS

1.In heights and distance problems, angles are always taken with respect to line of sight. (True/False)

In heights and distance problems, the primary angles (angle of elevation and angle of depression) are defined and measured with respect to the horizontal line (or ground level/eye level), not the line of sight, which is typically the hypotenuse of the right triangle formed.

2 The angle of elevation of top of a pole of height 30m from a point $30\sqrt{3}\,m$ is 60° (True/False)

3.Koresh observed a flower pot on the ground from the balcony of the first floor of a building at an angle of depression $\theta$, the height of the first floor of the building is p meters. Draw a rough diagram to this situation.




4.Define line of Sight.

The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

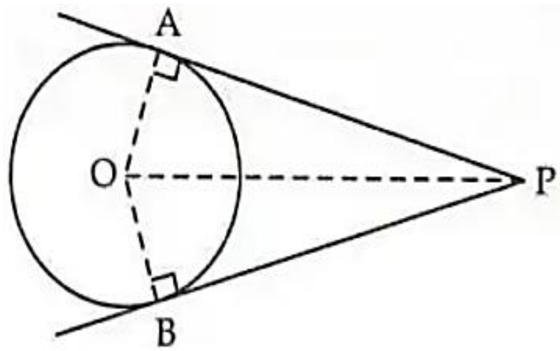
# 10.CIRCLES

**[7 Marks] [1 + 2 + 4 = 7 M]**

## LEVEL-1: RISING STAR

### 4 MARKS QUESTION ANSWERS

1. Prove that the length of the tangents drawn from an external point to a circle are equal.




Given: PA and PB tangents drawn to circle with centre O from an external point P

RTP: PA=PB

Construction: Join OA, OB and OP

Proof: $\angle OAP = \angle OBP = 90°$ ($\because$ Radius is perpendicular to tangent )

$In\ \Delta OAP\ and\ \Delta OBP$

OA= OB ($\because$ Radi of same circle )

OP = OP ($\because$ common side )

$\angle$OAP = $\angle$OBP ($\because$ each 90° )

$\therefore$ ΔOAP $\cong$ ΔOBP ($\because$ RHS Congruency )

$\therefore$ PA = PB ($\because$ CPCT )

Thus, the length of the tangents drawn from an external point to a circle are equal.

2. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD+BC.

We know that the length of tangents drawn from an external point of the circle are equal according to Theorem

The lengths of tangents drawn from an external point to a circle are equal

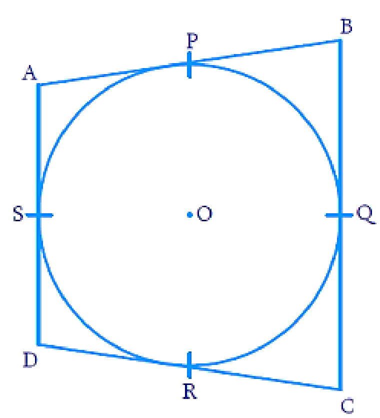
Therefore

AP=AS -----------i

BP=BQ-----------ii

CR=CQ-----------iii

DR=DS-----------iv




Adding all above then

AP + BP + CR + DR = AS + BQ + CQ + DS

On regrouping them

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

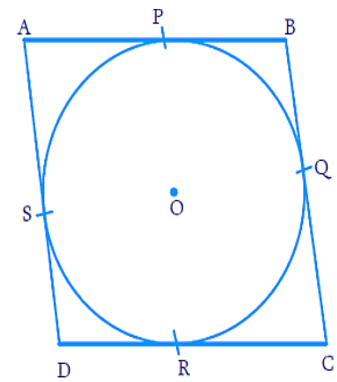
$\Longrightarrow$ AB + CD = AD + BC

$\therefore$ AB + CD = AD+BC.

3. Prove that the parallelogram circumscribing a circle is a rhombus.

We know that the length of tangents drawn from an external point of the circle are equal according to Theorem The lengths of tangents drawn from an external point to a circle are equal

Therefore




AP=AS -----------i

BP=BQ-----------ii

CR=CQ-----------iii

DR=DS-----------iv

Adding all above then

AP + BP + CR + DR = AS + BQ + CQ + DS

On regrouping them

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Longrightarrow AB + CD = AD + BC$ ----------(1)

ABCD is a parallelogram. Therefore, opposite sides are equal.

AB= CD, BC= AD ----------(2)

Now sub (2) in (1) we get

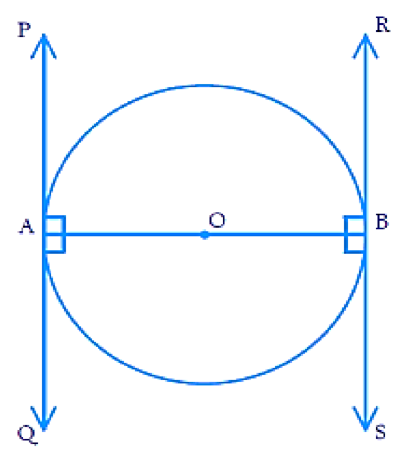
$AB + AB = BC + BC \Longrightarrow 2AB = 2BC \Longrightarrow AB = BC$

$\therefore AB = BC = CD = DA$

This implies that all four sides are equal

∴ the parallelogram circumscribing a circle is a rhombus

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.




Let AB be a diameter of the circle.

Two tangents PQ and RS are drawn at points A and B respectively.

A tangent to a circle is a line that intersects the circle at only one point.

From theorem the tangent at any point of a circle is perpendicular to the radius through the point of contact.

From the theorem Radius of a circle is perpendicular to the tangent at the point of contact

Thus, OA ⊥ PQ and OB ⊥ RS

∠PAO =$90^o$, ∠RBO= $90^o$

∠OAQ = $90^o$, ∠OBS = $90^o$

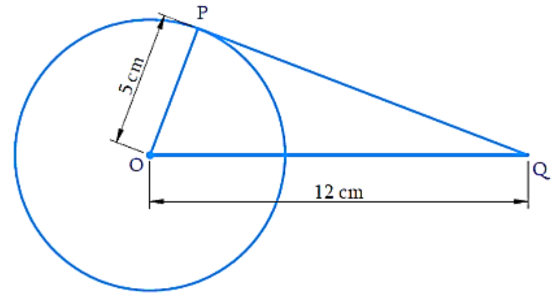
Here ∠OAQ, ∠OBR & and ∠PAO & ∠OBS are two pairs of alternate interior angles and they are equal.

If the Alternate interior angles are equal, then lines PQ and RS should be parallel. We know that PQ & RS are the tangents drawn to the circle at the ends of the diameter AB.

Hence, it is proved that tangents drawn at the ends of a diameter of a circle are parallel

## 2 MARKS QUESTION ANSWERS

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre 'O' at a point Q so that OQ = 12 cm. Find the length of PQ.




Given radius OP= 5 cm, OQ = 12cm

By Pythagoras theorem

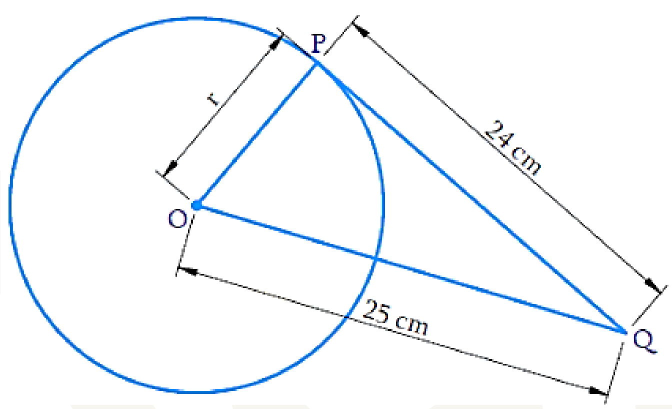
$OQ^2 = OP^2 + PQ^2 \Rightarrow 12^2 = 5^2 + PQ^2 \Rightarrow 144 = 25 + PQ^2$

$\Rightarrow PQ^2 = 144 - 25 \Rightarrow PQ^2 = 119 \Rightarrow PQ = \pm\sqrt{119}$

PQ is length of tangent and it cannot be negative

So, length of the tangent PQ= $\sqrt{119}$ cm

2.From a point Q, the length of the tangent to circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.




Given the length of the tangent to circle PQ= 24 cm

the distance of Q from the centre OQ= 25 cm

Tangent at any point of a circle is perpendicular to the radius through the point of contact

So ΔOPQ is a right-angled triangle

By using Pythagoras theorem

$OQ^2 = OP^2 + PQ^2 \Rightarrow 25^2 = OP^2 + 24^2$

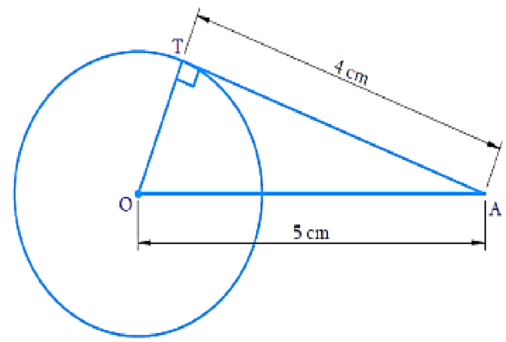
$\Rightarrow OP^2 = 25^2 - 24^2 \Rightarrow OP^2 = (25 + 24)(25 - 24) = 49 \times 1 \Rightarrow OP^2 = 49$

$\Rightarrow OP = \pm\sqrt{49} \Rightarrow OP = \pm 7$

OP is the radius of the circle cannot be negative

$\therefore$ the radius of the circle OP = 7cm

3. The length of the tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.




Given the length of the tangent to circle AT= 4 cm

the distance of Q from the centre $\quad OA = 5$ cm

Tangent at any point of a circle is perpendicular to the radius through the point of contact Therefore, $\angle$ OTA= $90^0$

So $\Delta$OTA is right angled triangle.

By using Pythagoras theorem

$$OA^2 = OT^2 + AT^2 \Rightarrow 5^2 = OT^2 + 4^2$$

$$\Rightarrow OT^2 = 5^2 - 4^2 \Rightarrow OT^2 = (5+4)(5-4) = 9 \times 1 \Rightarrow OT^2 = 9$$

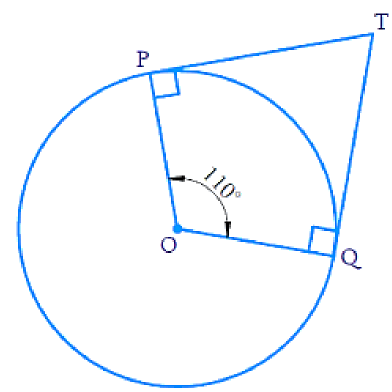
$$\Rightarrow OT = \pm\sqrt{9} \Rightarrow OT = \pm 3$$

OT is the radius of the circle and it cannot be negative

Hence the radius of the circle $OT = 3$ cm

4. If TP and TQ are the two tangents to a circle with centre 'O' so that $\angle POQ = 110°$, then find $\angle PTQ$.

(i) TP and TQ are tangents to a circle with Centre O $\quad$ (ii) $\angle$ POQ = $110^0$




In a quadrilateral, sum of four angles = 360°

therefore, in quadrilateral OPTQ

$$\angle OPT + \angle PTQ + \angle POQ + \angle OQT = 360° \Rightarrow 90° + \angle PTQ + 110° + 90° = 360°$$

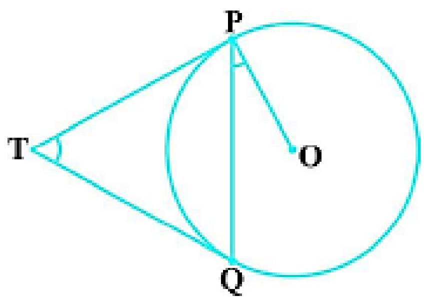
$$\Rightarrow 290° + \angle PTQ = 360° \quad \Rightarrow \angle PTQ = 360° - 290° = 70°$$

$$\therefore \angle PTQ = 70°$$

# LEVEL-2: SHINING STAR

## 4 MARKS QUESTION ANSWERS

1. Two tangents TP and TQ are drawn to a circle with centre 'O' from an external point T.
Prove that $\angle PTQ = 2\angle OPQ$




Solution: We are given a circle with centre O, an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact

we need to prove that $\angle PTQ = 2\angle OPQ$

Let $\angle OPQ = \theta$

then $\angle TPQ = 90° - \theta$ [ $\because$ The tangent at any point of a circle is perpendicular to the radius through the point of contact.]

Now, by Theorem: The lengths of tangents drawn from an external point to a circle are equal.

TP = TQ. So, TPQ is an isosceles triangle.

Therefore, $\angle TPQ = \angle TQP = 90° - \theta$

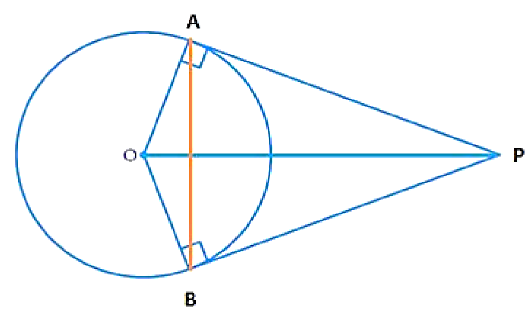
From angle sum property in $\Delta$TPQ

$\angle PTQ + \angle TPQ + \angle TQP = 180° \Rightarrow \angle PTQ + 90° - \theta + 90° - \theta = 180°$

$\Rightarrow \angle PTQ + 180° - 2\theta = 180° \Rightarrow \angle PTQ = 2\theta$

$\therefore \angle PTQ = 2\angle OPQ$ [ $\because \theta = \angle OPQ$ ]

2. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.




According to Theorem: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle OAP = \angle OBP = 90^0$ ................ (i)

In a quadrilateral, sum of four angles = 360°

therefore, in quadrilateral OAPB

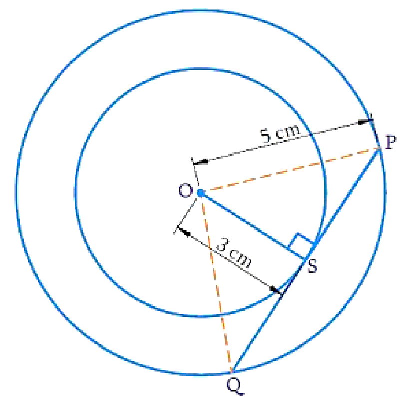
$\angle OAP + \angle APB + \angle OBP + \angle BOA = 360° \Rightarrow 90^0 + \angle APB + 90^0 + \angle BOA = 360°$

$\Rightarrow 180^0 + \angle APB + \angle BOA = 360° \Rightarrow \angle APB + \angle BOA = 360° - 180^0$

$\Rightarrow \angle APB + \angle BOA = 180^0$ where $\angle APB$=Angle between the two tangents PA and PB From an external point P , $\angle BOA$ =Angle subtended by the the line segment joining the points of contact at the centre.

hence the angle between the two tangents drawn from an external point to a circle is supplementary

3. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle with touches the smaller circle.




Two concentric circles are of radii 5cm and 3cm.
The length of the chord of the larger circle which touches the smaller circle.
Chord of the larger circle is a tangent to the smaller circle.
PQ is chord of a larger circle and tangent of a smaller circle.
Tangent is perpendicular to the radius at the point of contact

$\therefore \angle OSP = 90^0$

In Right angled $\Delta OSP$

By the Pythagoras Theorem,

$$OP^2 = OS^2 + SP^2 \Rightarrow 5^2 = 3^2 + SP^2 \Rightarrow 25 = 9 + SP^2$$

$$\Rightarrow SP^2 = 25 - 9 \Rightarrow SP^2 = 16 \Rightarrow SP = \pm\sqrt{16} \Rightarrow SP = \pm 4$$

SP is length of tangent and it cannot be negative

$\therefore$ SP= 4cm. QS = SP (Perpendicular from center bisects the chord considering the larger circles)

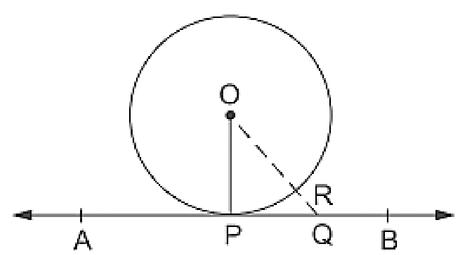
Therefore, QS= SP = 4 cm  Length of the chord PQ= QS + SP = 4 + 4 = 8 cm

Therefore, the length of the chord of the larger circle = 8cm.

## 2 MARKS QUESTION ANSWERS

1. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of a contact.

Solution:




Let a circle with centre O and a tangent AB at a point P of the circle.
RTP: OP$\perp$ *AB*.

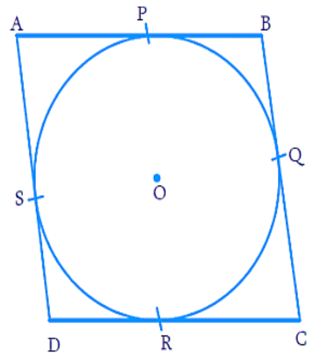
Take a point Q, other than P, on AB.
Q is a point on the tangent AB, other than the point of contact P.
$\therefore$ Q lies outside the circle.
Let OQ intersect the circle at R.
Then, OR<OQ [a part is less than the whole]......(i)
But, OP=OR [radii of the same circle].......(ii)
$\therefore$OP<OQ [from (i) and (ii)].
Thus, OP is shorter than any other line segment joining O to any point of AB, other than P.
That means OP is the shortest distance between point O and line AB.
But, the shortest distance between a point and a line is the perpendicular distance.
Hence, OP$\perp$AB .

2. Define tangent of a circle and secant of a circle.

Tangent: A line that touches the circle at only one point is called a tangent of a circle

Secant: A line that intersects the circle at two different points is called a Secant of a circle

3. A parallelogram ABCD is drawn to circumscribe a circle then write the relation between its sides.




For any quadrilateral circumscribing a circle, the lengths of tangents from a vertex to the circle are equal

$AP = AS,\ BP = BQ,\ CR = CQ,\ DR = DS$

Adding these tangent segments shows that the sum of opposite sides are equal:

$AB + CD = AD + BC$ -----(1 )

In a parallelogram, opposite sides are equal

$AB = CD,\ AD = BC$ ----- (2)

Substituting (2) in (1) then

$AB + AB = AD + AD$

$\Longrightarrow 2AB = 2AD,$

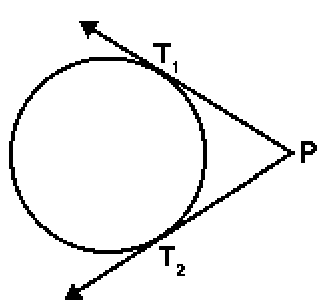
$AB = AD.$

Since $AB = AD$, and opposite sides are equal

it means that all four sides are equal ($AB = BC = CD = DA$), and the shape is a rhombus.

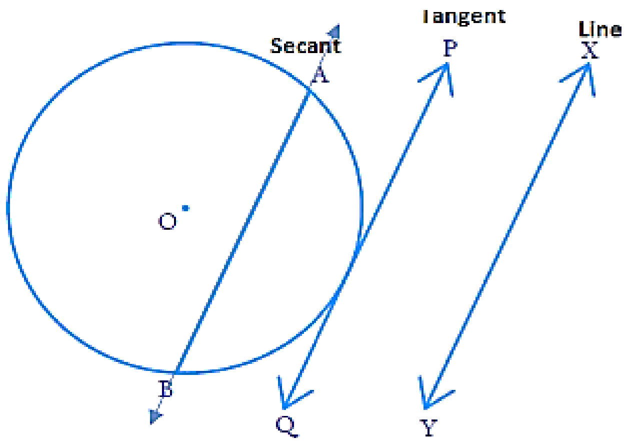
## 1 MARK QUESTION ANSWERS

1. Create a geometrical design involving two tangents drawn from an external point to the circle.




$T_1P$ and $T_2P$ are two tangents drawn from an external point P to the circle.

2.Draw a circle and two lines parallel to a given line such that one is a tangent and the other a secant to the circle.

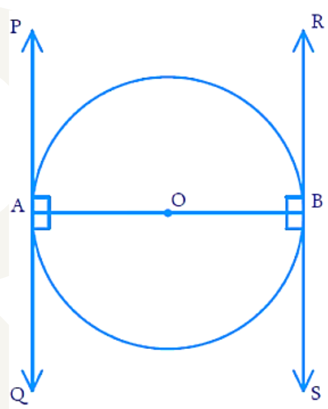



XY is the given line

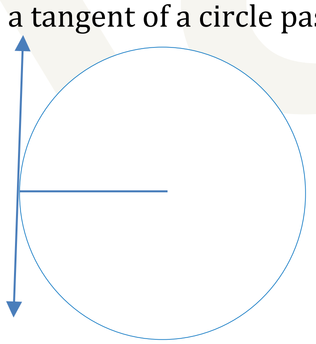
AB is the secant parallel to XY, AB ‖ XY

PQ is the tangent parallel to XY, PQ ‖ XY

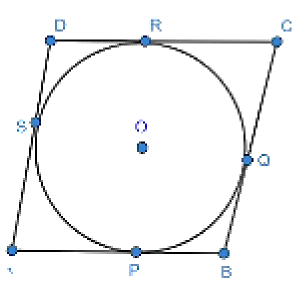
3. Generate a geometrical design that the tangents drawn at the ends of a diameter of a circle are parallel.



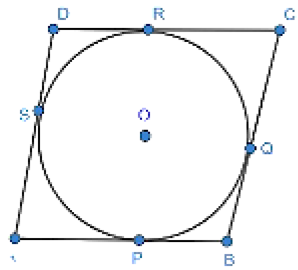

4. Draw a rough sketch as a tangent of a circle passes through the end point of its radius.



5. Draw a rough sketch of a parallelogram circumscribing a circle.



6. Draw a circle inscribing in a quadrilateral.




7. How many tangents can a circle have?

A circle can have infinitely many tangents

8. A circle can have........... parallel tangents at the most.

A)1 B) 2 ✓ C) 0 D) Infinite

9. The common point of a tangent to a circle and the circle is called.

point of contact.

10. A tangent to a circle intersects it the circle .......... points.

A tangent to a circle is a line that intersects the circle at only one point

7. How many tangents can a circle have?

A circle can have an infinite number of tangents

# 11.AREAS RELATED TO CIRCLES

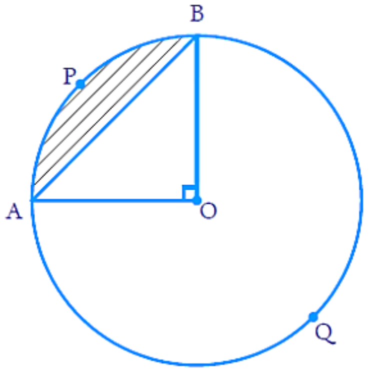
**[8 Marks] [8 M]**

## LEVEL-1: RISING STAR

### 8 MARKS QUESTION ANSWERS

1.A chord of a circle of radius 10 cm subtends a right angle at the Centre. Find the area of the corresponding

(i) Minor segment (ii) Major segment (use π = 3.14)




Solution: Given A chord of a circle of radius r = 10 cm

A chord of a circle subtends angle at the Centre $\theta = 90°$

i)Area of minor segment APB = Area of sector OAPB − Area of right ΔAOB

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} \times \text{OA} \times \text{OB}$$

$$= \frac{90°}{360°} \times \pi r^2 - \frac{1}{2} \times \text{r} \times \text{r}$$

$$= \frac{1}{4}\pi r^2 - \frac{1}{2} r^2$$

$$= r^2\left(\frac{1}{4}\pi - \frac{1}{2}\right)$$

$$= (10cm)^2 \times \left(\frac{\pi - 2}{4}\right)$$

$$= 100cm^2 \times \left(\frac{3.14-2}{4}\right)$$

$$= 100cm^2 \times \left(\frac{1.14}{4}\right)$$

$$= 25\ cm^2 \times 1.14 = 28.5\ cm^2$$

∴ Area of the minor segment = 28.5 $\text{cm}^2$

ii) Area of major segment AQB = $\pi r^2$ − Area of minor segment

$$= \pi r^2 - 28.5cm^2$$

$$= 3.14 \times (10cm)^2 - 28.5\ \text{cm}^2$$

$$= 314\ \text{cm}^2 - 28.5\ \text{cm}^2$$

$$= 285.5\ \text{cm}^2$$

∴ Area of the major segment = 285.5 $\text{cm}^2$

2.A chord of a circle of radius 12 cm subtends an angle of 120° at the Centre. Find the area of the corresponding segment of the circle. (use $\pi = 3.14$ and $\sqrt{3} = 1.732$)

Solution: Given Radius ( r ) = 12cm , $\theta = 120°$

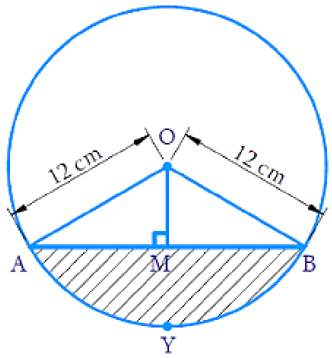
Area of sector OAYB $= \frac{120°}{360°} \times \pi r^2 = \frac{1}{3} \times 3.14 \times (12cm)^2$

$= \frac{1}{3} \times 3.14 \times 12\,cm \times 12\,cm$

$= 3.14 \times 4\,cm \times 12\,cm$

$= 150.72\text{cm}^2$

Draw a perpendicular chord OM from O to chord AB




In ΔAOM and ΔBOM

AO= BO= r (∵ radius of circle)

OM=OM (∵ common side)

∠OMA =∠OMB = 90°

∴ ΔAOM ≅ ΔBOM (∵ RHS congruence)

$\Rightarrow \angle AOM = \angle BOM$ (∵ CPCT )

$\therefore \angle AOM = \angle BOM = \frac{1}{2} \angle AOB = \frac{1}{2} \times 120° = 60°$

In right ΔAOM $\text{Sin}60° = \frac{AM}{OA} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{12} \Rightarrow AM = \frac{\sqrt{3}}{2} \times 12\,cm = 6\sqrt{3}\,cm$

$\Rightarrow AB = 2AM = 2 \times 6\sqrt{3}\,cm \Rightarrow AB = 12\sqrt{3}\,cm$

$\cos 60° = \frac{OM}{OA} \Rightarrow \frac{1}{2} = \frac{OM}{12} \Rightarrow OM = \frac{1}{2} \times 12\,cm = 6\,cm$

area of ΔAOB $= \frac{1}{2} \times AB \times OM = \frac{1}{2} \times 12\sqrt{3}\,cm \times 6\,cm$

$= 36\sqrt{3}\text{cm}^2 = 36 \times 1.73 = 62.28\text{cm}^2$

Area of segment AYB = Area of sector OAYB − Area of right ΔAOB

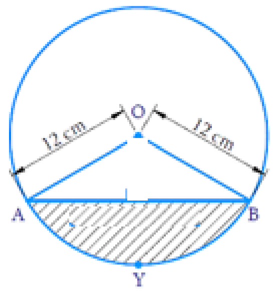
$= 150.72\text{cm}^2 - 62.28\text{cm}^2 = 88.44\text{cm}^2$

Area of the segment = 88.44 $\text{cm}^2$

## Alternate method

2.A chord of a circle of radius 12 cm subtends an angle of 120° at the Centre. Find the area of the corresponding segment of the circle. (use $\pi = 3.14$ and $\sqrt{3} = 1.732$)

Solution:




Given Radius, $r = 12\text{cm}$, angle $\theta = 120°$

i) Area of sector OAYB = $\frac{120°}{360°} \times \pi r^2$

$= \frac{120°}{360°} \times \frac{22}{7} \times 12^2 = \frac{1}{3} \times 3.14 \times 12 \times 12 = 48 \times 3.14 = 150.72 \text{ cm}^2$

ii)Area of the segment formed by the corresponding chord = Area of the sector AOYP –Area of the ΔAOB

Area of the sector AOYP = $150.72 \text{ cm}^2$

Area of ΔAOB $= \frac{1}{2} r^2 \sin\theta = \frac{1}{2} \times 12^2 \times \sin 120° = \frac{1}{2} \times 12 \times 12 \times \frac{\sqrt{3}}{2}$ $\qquad [\because \sin 120° = \frac{\sqrt{3}}{2}]$

$= 36\sqrt{3} \text{ cm}^2 = 36 \times 1.732 = 62.35 \text{cm}^2$

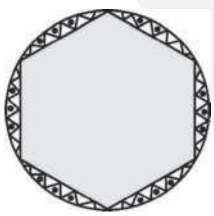
Area of the segment = $150.72 \text{ cm}^2 - 62.35 \text{cm}^2 = 88.37 \text{ cm}^2$

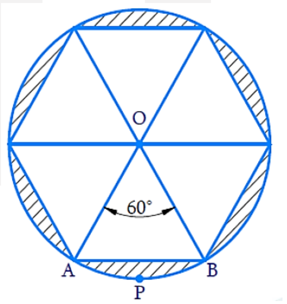
Area of the segment = $88.37 \text{ cm}^2$

## LEVEL-2: SHINING STAR

3.A round table cover has six equal designs as shown in figure. If the radius of the cover is 28 cm.

Find the cost of making the designs at the rate of ` 0.35 per cm² (use = 1.7)






Solution: From the figure we observe the designs are made in the segments of a circle.

Since the table cover has 6 equal designs

∴ angle subtended by each chord (bounding the segment) at the center $= \frac{360°}{6} = 60°$

Consider ∠OAB = ∠OBA ( $\because OB = OA$ , angles opposite equal sides in a triangle are equal )

From angle sum property ∠OAB + ∠AOB +∠OBA = 180° i.e ∠OAB + 60° +∠OBA = 180°

i.e. 2 ∠OAB + 60° = 180° i.e. 2 ∠OAB = 180° − 60°

i.e. 2∠OAB = 120° ∴ ∠OAB = 60°

∴ ΔAOB is an equilateral triangle

Area of ΔAOB $= \frac{\sqrt{3}}{4} \times \text{side}^2 = \frac{\sqrt{3}}{4} \times 28^2$

$= \frac{\sqrt{3}}{4} \times 28 \times 28 = 7 \times 28 \times \sqrt{3} = 196 \times 1.7 == 333.2 cm^2$

Area of sector OAPB = $\frac{60°}{360°} \times \pi\, r^2 = \frac{60°}{360°} \times \frac{22}{7} \times 28^2 = \frac{1}{6} \times \frac{22}{7} \times 28 \times 28 = \frac{11 \times 4 \times 28}{3} = \frac{1232}{3} cm^2$

Area of segment APB = Area of sector OAPB − Area of ΔAOB

Area of segment APB = $( \frac{1232}{3} - 333.2 )\, cm^2$

Area of designs = 6 Area of segment

Area of designs = $6 \times ( \frac{1232}{3} - 333.2 )\, cm^2 = 6 \times \frac{1232}{3} - 6 \times 333.2 )$

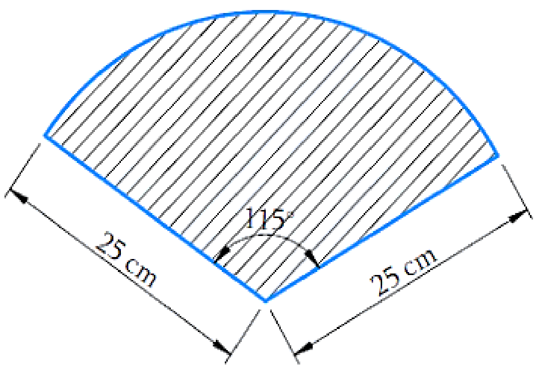
$= (2 \times 1232 - 6 \times 333.2) cm^2 = (2464 – 1999.2\, ) cm^2 = 464.8\, cm^2$

the cost of making of $1\, cm^2$ *designs* = ₹0.35

therefore, the cost of making of $464.8\, cm^2$ *designs* = ₹0.35 × 464.8 = ₹162.68

The cost of making the designs = ₹162.68

4.A car has two wiper which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.




Solution: Area cleaned at the sweep of blades of each wiper

= Area of the sector of a circle with radius 25 cm and of angle 115°

$= \frac{\theta}{360°} \times \pi r^2 = \frac{115°}{360°} \times \pi \times 25 \times 25 = \frac{23}{72} \times 625\pi$

Since there are 2 identical blade length wipers

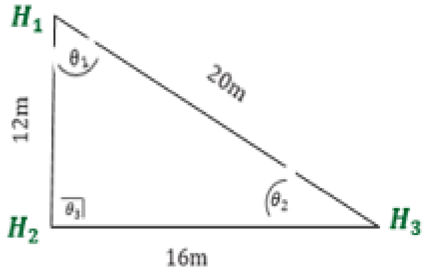
Total area cleaned at each sweep of the blades = $2 \times \frac{23}{72} \times 625\pi = 2 \times \frac{23}{72} \times 625 \times \frac{22}{7}$

$= \frac{23 \times 11 \times 625}{18 \times 7} = \frac{158125}{126}\ \text{cm}^2 = 1254.96\ \text{cm}^2$

Total area cleaned at each sweep of the blades = $1254.96 cm^2$

5.Three horses are tethered with 7 m ropes at three corner of a triangular field having sides

12 m, 16 m, 20 m. Find the area of the field that can be grazed by the horses.




Solution:

First, we check if the given sides (12 m, 16 m, 20 m) form a right-angled triangle using the Pythagorean theorem,

$12^2 + 16^2 = 144 + 256 = 400 = 20^2$ the field is a right-angled triangle

The field is a triangle with side lengths 12 m, 16 m, and 20 m.

The sum of the interior angles of any triangle is always 180°

Each horse is tethered at a corner with a 7 m rope, allowing it to graze a circular sector within the field boundaries.

The total area grazed is the sum of the areas of the three sectors.

The area of a sector $= \frac{\theta}{360°} \times \pi r^2$

Using the rope length r=7 m

The total area grazed = sum of the areas of the three sectors A = . $\frac{\theta_1}{360°} \times \pi r^2 + \frac{\theta_2}{360°} \times \pi r^2 + \frac{\theta_3}{360°} \times \pi r^2$

$$\Rightarrow A = \frac{\pi r^2}{360°} [\theta_1 + \theta_2 + \theta_3]$$

$$= \frac{\pi r^2}{360°} [180°]$$

$$= \frac{\pi r^2}{2} = \frac{22}{7} \times 7^2$$

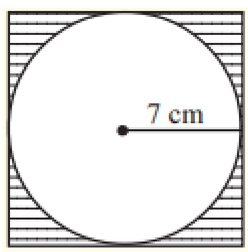
$$= \frac{\frac{22}{7} \times 7^2}{2}$$

$$= \frac{22 \times 7}{2}$$

$$= 11 \times 7 = 77$$

The area of the field that can be grazed by the horses = 77 $m^2$

6.A circle with 7 cm radius inscribed in a square. Find the area of shaded region.




Solution: The radius of the inscribed circle is r=7 cm

When a circle is inscribed in a square,

the diameter of the circle is equal to the side length of the square

The side length of the square, is calculated as $s = 2 \times r = 2 \times 7 = 14\ cm$

The area of the square $A_{square} = s^2$

$$= (14cm)^2$$

$$= 196\ cm^2$$

The area of the circle $A_{circle} = \pi r^2$

$= \pi (7cm)^2$

$= \frac{22}{7} \times 7 \times 7$

$= 22 \times 7\ cm^2$

$= 154\ cm^2$

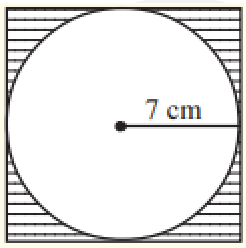
The area of shaded region = $A_{square} - A_{circle}$

$= 196\ cm^2 - 154\ cm^2 = 42\ cm^2$

The area of shaded region $= 42\ cm^2$

## Alternate method

6.A circle with 7 cm radius inscribed in a square. Find the area of shaded region.




Solution:

The radius of the inscribed circle is r=7 cm

When a circle is inscribed in a square,

the diameter of the circle is equal to the side length of the square.

The side length of the square, is calculated as $s = 2 \times r$= 2r

The area of shaded region = $A_{square} - A_{circle}$

$= 4r^2 - \pi r^2$ [ ∵ The area of the square $= s^2 = (2r)^2 = 4r^2$ ]

$= r^2(4 - \pi) = r^2\left(4 - \frac{22}{7}\right) = r^2\left(\frac{28-22}{7}\right) = r^2\left(\frac{6}{7}\right)$

$= 7^2\left(\frac{6}{7}\right) = 7 \times 7 \times \frac{6}{7} = 6 \times 7 = 42\ cm^2$

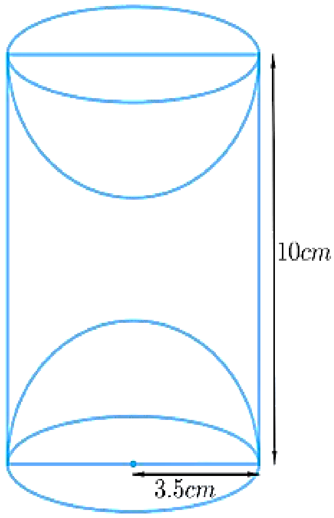
# 12 SURFACE AREAS AND VOLUMES

[7 Marks] [1 + 2 + 4 = 7 M]

## LEVEL-1: RISING STAR

### 4 MARKS QUESTION ANSWERS

1. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article?




Solution: Given Height of the cylinder(h) = 10cm

Radius of the cylinder (r)= Radius of the hemisphere(r) = 3.5cm

TSA of the article =

$2 \times$ CSA of the hemispherical part + CSA of thecylindrical part

$= 2 \times 2\,\pi r^2 + 2\pi rh = 2\,\pi r\,(2r + h)$

$= 2 \times \frac{22}{7} \times 3.5\text{ cm}\,(2 \times 3.5\text{cm} + 10\text{cm})$

$= 2 \times 22 \times 0.5\text{ cm}\,(7\text{cm} + 10\text{cm})$

$= 1.0\text{ cm} \times 22\text{cm} \times 17\text{ cm}$

$= 22\text{cm} \times 17\text{ cm} = 374\text{cm}^2$

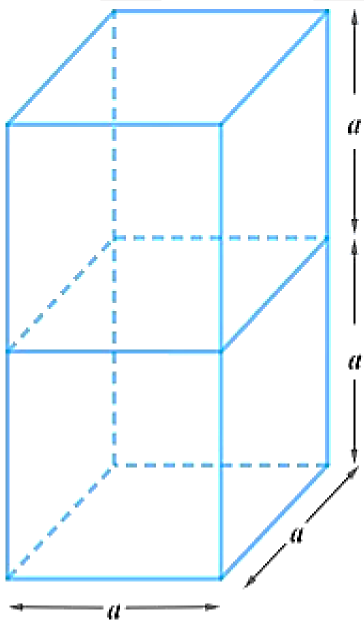
$\therefore$ total surface area of the article $= 374cm^2$

2. 2 cubes each of volume 64 $cm^3$ are joined end to end. Find the surface area of the resulting cuboid?

Solution: Let the length of the edge of each cube is $a$

Therefore, volume of the cube = $a^3$

Given cube volume of cube V = 64 $cm^3$




$a^3 = 64\ \ 64\text{cm}^3 = (4\text{cm})^3$

$\Longrightarrow a = 4\ cm$

from the sum 2 cubes are joined end to end then it forms a cuboid

Therefore,

length of the resulting cuboid, $l = a = 4cm$

breadth of the resulting cuboid, $b = a = 4cm$

height of the resulting cuboid, $h = 2a = 2 \times 4cm = 8cm.$

surface area of the resulting cuboid $= 2(\,lb + bh + lh)$

$= 2(4\text{cm} \times 4\text{cm} + 4\text{cm} \times 8\text{cm} + 4\text{cm} \times 8\text{cm})$

$= 2(16cm^2 + 32cm^2 + 32cm^2) = 2 \times 80cm^2 = 160cm^2$

$\therefore$ surface area of the resulting cuboid $= 160\text{cm}^2$

## 2 MARKS QUESTION ANSWERS

1. Consider the following situations. In each find out whether you need volume or area and why?

i) Quantity of water inside a bottle. I) VOLUME: 3-D SHAPE

ii) Canvas needed for making a tent. AREA: L.S.A OR T.S.A.

iii) Number of bags inside the lorry. VOLUME: 3-D SHAPE

iv) Number of match sticks that can be put in the match box. IV) VOLUME: 3-D SHAPE

2. Find the T.S.A of a right circular cylinder of radius 7 cm and height 10 cm?

Solution: Radius of Cylinder (r) = 7 cm

Height of Cylinder(h) = 10 cm

TSA of Cylinder= $2\pi r(h+r)$

$\Longrightarrow$ $2 \times \frac{22}{7} \times 7\ (10+7) = 44 \times 17$

TSA of Cylinder= 748 $cm^2$

3. Find the volume of a right circular cone of radius 6 cm and height 7 cm?

Solution: Radius of Cone (r) = 6 cm

Height of Cone (h) = 7 cm

Volume = $\frac{1}{3}\pi r^2 h$

Volume $= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7 = 264$ $cm^3$

4. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

Solution: Volume of the cylinder $V_1 = \pi r^2 h$

Volume of the cone $V_2 = \frac{1}{3}\pi r^2 h$

Volume of the hemisphere $V_3 = \frac{2}{3}\pi r^3$

Volume of the cylinder : Volume of the cone : Volume of the hemisphere = $V_1 : V_2 : V_3$

Substituting the formulas $V_1 : V_2 : V_3 = \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3$

$= h : \frac{1}{3} h : \frac{2}{3} r = 1 : \frac{1}{3} h : \frac{2}{3} h = 1 : \frac{1}{3} : \frac{2}{3} = 3 : 1 : 2$

ratio of the volumes of cylinder, cone and hemisphere = 3: 1 : 2

5. Find the volume and the total surface area of a hemisphere of radius 3.5 cm.

Solution: Radius of the hemisphere = 3.5 cm

volume of the hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (3.5)^3$

$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$

$= \frac{2}{3} \times 22 \times 0.5 \times 3.5 \times 3.5$

$= \frac{2}{3} \times 11.0 \times 12.25$

$= \frac{11 \times 24.5}{3} = \frac{269.5}{3} = 89.83 cm^3$

$\therefore$ volume of the hemisphere = $89.83 cm^3$

TSA of the hemisphere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times (3.5)^2$$

$$= 3 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 3 \times 22 \times 0.5 \times 3.5$$

$$= 3 \times 11.0 \times 3.5$$

$$= 33 \times 3.5 = 115.5 \text{ cm}^2$$

$\therefore$ total surface area of a hemisphere $= 115.5 \text{ cm}^2$

## 1 MARK QUESTION ANSWERS

1. What is the curved surface area of a cone with radius 'r' and slant height 'l'? (A)
   $A)\ \pi rl \quad B)\ \pi r(l+r) \quad C)\ \frac{1}{3}\pi r^2 h \quad D)\ \pi r^2$

2. The volume of a right circular cylinder with base radius 'r' and height 'h' is......... (B)
   $A)\ 2\pi rh$ B) $\pi r^2 h$ $C)\ 2\pi r(r+h)$ $D)\ \frac{1}{3}\pi r^2 h$

3. Volume of cube is 125 $cm^3$ then its side ............ (D)
   (A) 6 cm B) 12 cm C) 10 cm D) 5 cm

4. Identify the correct relation. (C)

p. C. S. A of Hemisphere — i) $\frac{2}{3}\pi r^3$

q. T. S. A of Hemisphere — ii) $2\pi r^2$

r. Volume of Hemisphere — iii) $3\pi r^2$

| A) | B) | C) | D) |
|---|---|---|---|
| p → (i) | p → (iii) | p → (ii) | p → (i) |
| q → (ii) | q → (ii) | q → (iii) | q → (iii) |
| r → (iii) | r → (i) | r → (i) | r → (ii) |

5. Identify the correct relation. (B)

p. LSA of cylinder — i) $\pi r^2 h$

q. TSA of cylinder — ii) $2\pi rh$

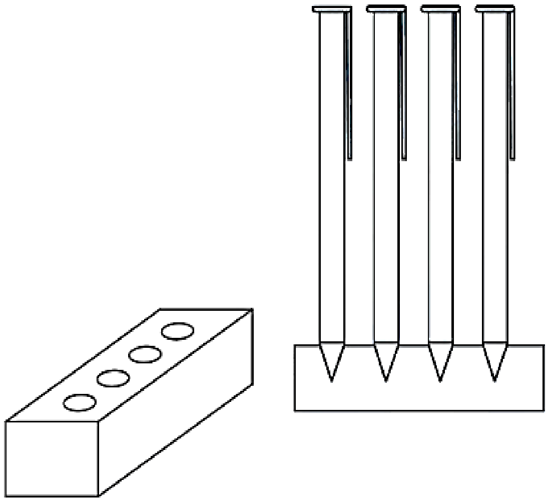
r. Volume of cylinder — iii) $2\pi r(r+h)$

| A) | B) | C) | D) |
|---|---|---|---|
| p → (i) | p → (ii) | p → (iii) | p → (i) |
| q → (ii) | q → (iii) | q → (ii) | q → (iii) |
| r → (iii) | r → (i) | r → (i) | r → (ii) |

## LEVEL-2: SHINING STAR

### 4 MARKS QUESTION ANSWERS

1. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depression is 0.5 cm and the depth is 1.40 cm. Find the volume of wood in the entire stand.



Solution: Depth of each conical depression $h_1 = 1.40\text{cm}$

Radius of each conical depression $r = 0.5$ cm

dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm

volume of the wood in the entire stand

$= \text{volume of the wooden cuboid} - 4 \times \text{ volume of the conical depressions}$

$= lbh - 4 \times \frac{1}{3}\pi r^2 h$

$= (15\text{cm} \times 10\text{cm} \times 3.5\text{cm}) - 4 \times \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4\text{cm}$

$= 525\text{cm}^3 - 4 \times \frac{1}{3} \times 22 \times 0.5 \times 0.5 \times 0.2\text{cm}$

$= 525\text{cm}^3 - 4 \times \frac{1}{3} \times 11cm^2 \times 0.1\text{cm}$

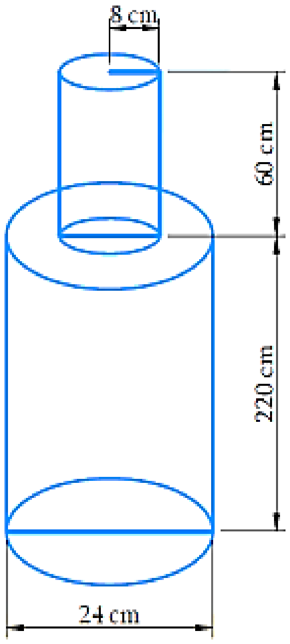
$= 525\text{cm}^3 - 4 \times \frac{1}{3} \times 1.1\text{cm}^3$

$= 525\text{cm}^3 - \frac{4.4}{3}\text{cm}^3$

$= 525\text{cm}^3 - 1.47\text{ cm}^3 = 523.53\text{ cm}^3$

$\therefore$ the volume of wood in the entire stand $= 523.53\text{ cm}^3$

2. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that $1\ cm^3$ of iron has approximately 8 g mass.




Solution: Diameter of larger cylinder $D = 24$ cm

Radius of larger cylinder $R = \frac{D}{2} = \frac{24}{2} = 12cm$

Height of larger cylinder $H = 220$ cm

Radius of smaller cylinder $= 8cm$

Height of smaller cylinder $h = 60$cm

volume of solid cylinder = volume of larger cylinder + volume of larger cylinder

$$= \pi R^2 H + \pi r^2 h = \pi( R^2 H + r^2 h )$$

$$= 3.14\ [\ (12cm)^2 \times 220cm + (8cm)^2 \times 60cm]$$

$$= 3.14\ [\ 144cm^2 \times 220cm + 64cm^2 \times 60cm]$$

$$= 3.14\ [\ 31680\ \text{cm}^3 + 3840\ \text{cm}^3]$$

$$= 3.14 \times 35520\text{cm}^3 = 111532.8\text{cm}^3$$

volume of solid cylinder=$111532.8\text{cm}^3$

Given mass of $1\text{cm}^3$ iron has approximately 8 g

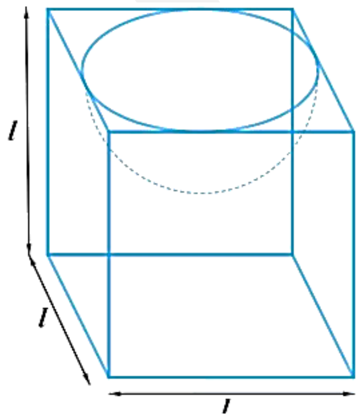
mass of the iron pole=8g × volume of solid iron pole

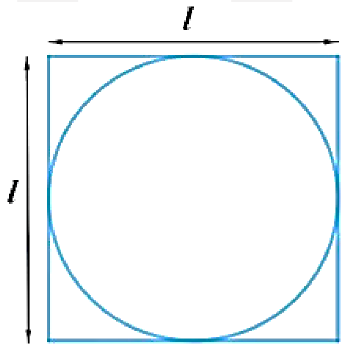
$$= 8\text{g} \times 111532.8\text{cm}^3 = 892262.4g$$

$$= \frac{892262.4}{1000}\text{kg} = 892.2624\text{kg} = 892.2624\text{kg}$$

$\therefore$ Mass of the pole = 892.26 kg

3. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter $l$ of the hemisphere is equal to the edge of the cube. Compute the surface area of the remaining solid.



top view

Solution: Diameter of the hemisphere = length the edge of the cube = $l$

Radius of the hemisphere $r = \frac{l}{2}$

surface area of the remaining solid =

TSA of the cubical part + CSA of the hemispherical part − Area of the base of the hemispherical part

$$= 6l^2 + 2\,\pi r^2 - \pi r^2 = 6l^2 + \pi r^2 = 6l^2 + \pi\left(\frac{l}{2}\right)^2$$

$$= 6l^2 + \pi\frac{l^2}{4} = l^2\left(6 + \frac{\pi}{4}\right)$$

$$= l^2\left(\frac{24+\pi}{4}\right) = \frac{l^2}{4}(\pi + 24)$$

$\therefore$ the surface area of the remaining solid $= \frac{l^2}{4}(\pi + 24) sq.units$

4. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of $\pi$

Solution: Radius of the hemispherical part = Radius of the conical part = $r = 1cm$

Height of the conical part $= h = r = 1cm$

volume of the solid = volume of the conical part + volume of the hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

Now substitute $h =$ 1cm and $r =$ 1cm in above then

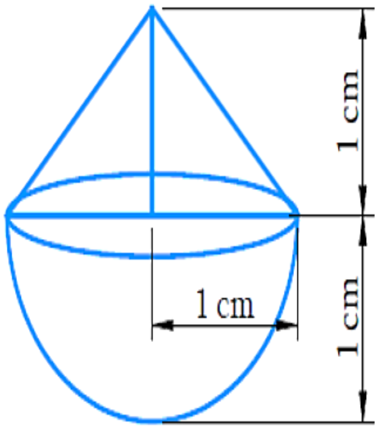
$$= \frac{1}{3}\pi(1cm)^2 \times 1cm + \frac{2}{3}\pi(1cm)^3$$

$$= \frac{1}{3}\pi \text{cm}^3 + \frac{2}{3}\pi \text{cm}^3$$

$$= \pi\left(\frac{1}{3} + \frac{2}{3}\right)\text{cm}^3$$

$$= \pi\left(\frac{1+2}{3}\right)\text{cm}^3$$

$$= \pi\left(\frac{3}{3}\right)\text{cm}^3 = \pi \text{cm}^3$$




$\therefore$ volume of the solid in terms of $\pi = \pi \text{cm}^3$

## 2 MARKS QUESTION ANSWERS

1. The radius of a sphere is r cm. It is divided into two equal parts. Find the whole surface area of two parts?

Solution: Given the radius of the sphere = r cm.

When the sphere is divided into two equal parts, each part becomes a hemisphere with same radius (r).

Total surface area of the hemisphere $= 3\pi r^2$

This includes the curved surface area and the area of the circular base.

Total surface area of two hemispheres $= 2 \times 3\pi r^2 = 6\pi r^2 cm^2$

$\therefore$ the whole surface area of two parts $= 6\pi cm^2$ .

2. Three solid metallic spherical balls of radii 3 cm, 4 cm and 5 cm are melted into a single spherical ball. Find its radius?

Solution: Radius of 1st sphere, $r_1 = 3cm$,

Radius of 2nd sphere, $r_2 = 4cm$,

Radius of 3rd sphere, $r_3 = 5cm$,

Let the radius of the resulting sphere be $r$.

Volume of the resulting sphere V = sum of the Volume of the three spheres

$$V = V_1 + V_2 + V_3$$

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{4}{3}\pi {r_1}^3 + \frac{4}{3}\pi {r_1}^3 + \frac{4}{3}\pi {r_1}^3$$

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\ ({r_1}^3 + {r_2}^3 + {r_3}^3)$$

$$\Rightarrow r^3 = ({r_1}^3 + {r_2}^3 + {r_3}^3)$$

Now substitute $r_1 =$ 3cm, $r_2 =$ 4cm and $r_3 =$ 5cm, in above then

$$r^3 = [3^3 + 4^3 + 5^3]$$

$$r^3 = 27 + 64 + 125$$

$$r^3 = 216 = 6^3$$

$$\therefore r = 6 \text{ cm}$$

Therefore, the radius of the sphere so formed will be 6 cm.

3. If the volume and surface area of sphere are numerically equal then find its radius?

Solution: The volume V of a sphere V= $\frac{4}{3}\pi r^3$

and the surface area A= $4\pi r^2$

If the volume and surface area of sphere are numerically equal then

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\Longrightarrow \frac{1}{3} r = 1$$

$$\Longrightarrow r = 3cm$$

Therefore, radius r = 3cm

## 1 MARK QUESTION ANSWERS

1. Statement - I: If radii of two spheres are in the ratio 2:3, then surface areas are in the ratio 4:9.

Statement- II: Volume of sphere is $\frac{2}{3}\pi r^3$ ( B)

A) S-I is true, S-II is true B) S-I is true, S-II is false✓

C) S-I is false, S-II is true D) S-I is false, S-II is false

2.Assertion(A): The maximum volume of a cone that can be cut from a cylinder of volume 120 $cm^3$ is 40 $cm^3$?

Reason (R): The volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same base and height?

A) Both A and R are true and R is the correct explanation of A. ✓ ( A )

B) Both A and R are true and but 'R' is not the correct explanation of A

C) A is true, but R is false D) A is false, but R is true.

# 13.STATISTICS

**12 Marks] [4 + 8 = 12 M**

**LEVEL-1: RISING STAR**

## 8 MARKS QUESTION ANSWERS

1.The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

| Literacy rate (in %) | 45-55 | 55-65 | 65-75 | 75-85 | 85-95 |
|---|---|---|---|---|---|
| No. of cities | 3 | 10 | 11 | 8 | 3 |

Solution: We know that, Class mark, $x_i = \frac{\text{Upper class limit+ Upper class limit}}{2}$

Class size, h = 10, Taking assumed mean, $a = 70$

| Literacy rate (in %) | No. of cities ($f_i$) | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_i u_i$ |
|---|---|---|---|---|
| 45-55 | 3 | 50 | $-2$ | $-6$ |
| 55-65 | 10 | 60 | $-1$ | $-10$ |
| 65-75 | 11 | 70 ($a$) | 0 | 0 |
| 75-85 | 8 | 80 | 1 | 8 |
| 85-95 | 3 | 90 | 2 | 6 |
| Total | $\sum f_i = 35$ | | | $\sum f_i u_i = -2$ |

Step deviation method:

From the table, we obtain $\sum f_i = 35, \quad \sum f_i u_i = -2$

$$\text{Mean}, (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\Rightarrow \text{Mean}, (\bar{x}) = 70 + \left(\frac{-2}{35}\right) \times 10$$

$$= 70 + \left(\frac{-2}{35}\right) \times 10 = 70 + \left(\frac{-4}{7}\right)$$

$$= 70 - 0.57 = 69.43\%$$

$\therefore$ The mean literacy rate=69.43%

$$7 \overline{)\ 4\ } \ (0.57$$
0
40
35
50
49
1

$\frac{-4}{7} = 0.57$

2.30 women were examined in a hospital by a doctor and the number of heart beats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method

| No. of heart beats/minute | 65-68 | 68-71 | 71-74 | 74-77 | 77-80 | 80-83 | 83-86 |
|---|---|---|---|---|---|---|---|
| No. of workers | 2 | 4 | 3 | 8 | 7 | 4 | 2 |

Solution: We know that, Class mark, $x_i = \frac{\text{Upper class limit+ Upper class limit}}{2}$

Class size, h = 3 Taking assumed mean, $a = 75.5$

| No. of heart beats/minute | No. of workers $(f_i)$ | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_i u_i$ |
|---|---|---|---|---|
| 65-68 | 2 | 66.5 | $-3$ | $-6$ |
| 68-71 | 4 | 69.5 | $-2$ | $-8$ |
| 71-74 | 3 | 72.5 | $-1$ | $-3$ |
| 74-77 | 8 | 75.5 $(a)$ | 0 | 0 |
| 77-80 | 7 | 78.5 | 1 | 7 |
| 80-83 | 4 | 81.5 | 2 | 8 |
| 83-86 | 2 | 84.5 | 3 | 6 |
| Total | $\sum f_i = 30$ | | | $\sum f_i u_i = 4$ |

Step deviation method:

From the table, we obtain $\sum f_i = 30$, $\sum f_i u_i = 4$

$$\text{Mean}, (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\Rightarrow \text{Mean}, (\bar{x}) = 75.5 + \left(\frac{4}{30}\right) \times 3$$

$$= 75.5 + \left(\frac{4}{\cancel{30}_{10}}\right) \times \cancel{3}^{1}$$

$$= 75.5 + \left(\frac{4}{10}\right)$$

$$= 75.5 + 0.4 = 75.9$$

∴ The mean heartbeats per minute for these women = 75.9

3.Consider the following distribution of daily wages of 50 workers of a factory.

| Daily wages (₹) | 500-520 | 520-540 | 540-560 | 560-580 | 580-600 |
|---|---|---|---|---|---|
| No of workers | 12 | 14 | 8 | 6 | 10 |

Find the mean daily wages of the workers of the factory by using an appropriate method.

Solution: We know that, Class mark , $x_i = \frac{\text{Upper class limit+ Upper class limit}}{2}$

Class size, h = 20, Taking assumed mean, $a = 550$

| Daily wages (₹) | No of workers ($f_i$ ) | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_i u_i$ |
|---|---|---|---|---|
| 500-520 | 12 | 510 | $-2$ | $-24$ |
| 520-540 | 14 | 530 | $-1$ | $-14$ |
| 540-560 | 8 | 550 ($a$) | 0 | 0 |
| 560-580 | 6 | 570 | 1 | 6 |
| 580-600 | 10 | 590 | 2 | 20 |
| Total | $\sum f_i = 50$ | | | $\sum f_i u_i = -12$ |

Step deviation method:

From the table, we obtain $\sum f_i = 50, \quad \sum f_i u_i = -12$

$$\text{Mean}, (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\Rightarrow \text{Mean}, (\bar{x}) = 550 + \left(\frac{-12}{50}\right) \times 20$$

$$= 550 + \left(\frac{-12}{50}\right) \times 20$$

$$= 550 + \left(\frac{-12}{50}\right) \times 20$$

$$= 550 - \frac{24}{5} = 550 - \frac{24}{5} \times \frac{2}{2}$$

$$= 550 - \frac{48}{10} = 550 - 4.8 = 545.2$$

∴ The mean daily wages of the workers of the factory = 545.2

4.The table below shows the daily expenditure on food of 25 households in a locality.

| Daily expenditure | 100-150 | 150-200 | 200-250 | 250-300 | 300-350 |
|---|---|---|---|---|---|
| No. of households | 4 | 5 | 12 | 2 | 2 |

Find the mean daily expenditure on food by a suitable method.

Solution: We know that, Class mark , $x_i = \frac{\text{Upper class limit+ Upper class limit}}{2}$

Class size, h = 50 , Taking assumed mean, $a = 225$

| Daily expenditure (in ₹) | Number of Households ($f_i$) | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_i u_i$ |
|---|---|---|---|---|
| 100-150 | 4 | 125 | −2 | −8 |
| 150-200 | 5 | 175 | −1 | −5 |
| 200-250 | 12 | 225 (a) | 0 | 0 |
| 250-300 | 2 | 275 | 1 | 2 |
| 300-350 | 2 | 325 | 2 | 4 |
| Total | $\sum f_i = 25$ | | | $\sum f_i u_i = -7$ |

From the table, we obtain $\sum f_I = 25, \quad \sum f_i u_I = -7$

$$\text{Mean}, (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\Rightarrow \bar{x} = 225 + \left(\frac{-7}{25}\right) \times 50 = 225 + \left(\frac{-7}{\cancel{25}_1}\right) \times \cancel{50}^2$$

$$\Rightarrow \bar{x} = 225 - 14 = 211$$

$$\therefore \bar{x} = ₹\,211.$$

Thus, the mean daily expenditure on food is ₹ 211.

5.The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

| Weight (in kg) | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
|---|---|---|---|---|---|---|---|
| Number of students | 2 | 3 | 8 | 6 | 6 | 3 | 2 |

Solution:

| Weight (in kg) | No. of students ($f_i$) | Cumulative frequency (c f) |
|---|---|---|
| 40-45 | 2 | 2 |
| 45-50 | 3 | $2 + 3 = 5$ |
| 50-55 | 8 | $5 + 8 = 13$ (c f ) |
| 55-60 | 6 ← $f$ | $13 + 6 = 19$ Median class |
| 60-65 | 6 | $19 + 6 = 25$ |
| 65-70 | 3 | $25 + 3 = 28$ |
| 70-75 | 2 | $28 + 2 = 30$ |
| Total | $n = 30$ | |

From the table, it can be observed that $n = 30 \Rightarrow \frac{n}{2} = \frac{30}{2} = 15$

Cumulative frequency (c f) just greater than 15 is 19, belonging to class 55 – 60.

Therefore, median class = 55 – 60

Class size, $h = 5$

Lower limit of the median class, $l = 55$

Frequency of the median class, f = 6

Cumulative frequency of class preceding median class, cf = 13

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h \Rightarrow \text{Median} = 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$\Rightarrow \text{Median} = 55 + \left(\frac{2}{6}\right) \times 5 \Rightarrow \text{Median} = 55 + \frac{5}{3}$$

$$\Rightarrow \text{Median} = 55 + 1.67 = 56.67$$

∴ The median weight of the students = 56.67 kg.

6.The following data gives the information on the observed lifetimes (in hours) of 225 electrical components.

| Life times (in hrs.) | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 |
|---|---|---|---|---|---|---|
| Frequency | 10 | 35 | 52 | 61 | 38 | 29 |

Determine the median lifetimes of the components.

Solution:

| Life times (in hrs.) | Frequency($f_i$) | Cumulative frequency (c f) |
|---|---|---|
| 0-20 | 10 | 10 |
| 20-40 | 35 | 10+ 35 = 45 |
| 40-60 | 52 | 45 + 52= 97 ( c f ) |
| 60-80 | 61 ← f | 97 + 61 = 158 Median class |
| 80-100 | 38 | 158 + 38= 196 |
| 100-120 | 29 | 196+ 29 = 225 |
| Total | n = 225 | |

From the table, it can be observed that n = 225 $\Rightarrow \frac{n}{2} = \frac{225}{2} = 112.5$

Cumulative frequency (c f) just greater than 112.5 is 158, belonging to class 60– 80.

Therefore, median class = 60– 80 , Class size, h = 20

Lower limit of the median class, $l$ = 60, Frequency of the median class, f = 61

Cumulative frequency of class preceding median class, cf = 97

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h \Rightarrow \text{Median} = 60 + \left(\frac{112.5 - 97}{61}\right) \times 20$$

$$\Rightarrow \text{Median} = 60 + \left(\frac{15.5}{61}\right) \times 20 = 60 + \left(\frac{15.5}{610}\right) \times 20 = 60 + \frac{310}{61}$$

Median = 60 + 5.08 = 65.08 Median = 65.08 hrs (approximately )

∴ The median lifetimes of the components = 65.08 hrs (approximately )

61 ) 310 ( 5.08
305
50
0
500
488
12

$\frac{310}{61} = 5.08$

## LEVEL-2: SHINING STAR

### 8 MARKS QUESTION ANSWERS

1.A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

| Number of cars | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 7 | 14 | 13 | 12 | 20 | 11 | 15 | 8 |

Solution: From the table, it can be observed that the maximum class frequency is 20, belonging to class interval $40-50$

Therefore, Modal class = $40-50$

Class size, h =10

Lower limit of modal class, $l = 40$

Frequency of modal class, $f_1 = 20$

Frequency of class preceding modal class, $f_0 = 12$

Frequency of class succeeding the modal class, $f_2 = 11$

| No. of cars | Frequency |
|---|---|
| 0-10 | 7 |
| 10-20 | 14 |
| 20-30 | 13 |
| 30-40 | 12 ← $f_0$ |
| 40-50 | 20 ← $f_1$ Modal class |
| 50-60 | 11 ← $f_2$ |
| 60-70 | 15 |
| 70-80 | 8 |

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \text{Mode} = 40 + \left(\frac{20 - 12}{2\times20 - 12 - 11}\right) \times 10$$

$$\Rightarrow \text{Mode} = 40 + \left(\frac{8}{40 - 23}\right) \times 10$$

$$\Rightarrow \text{Mode} = 40 + \left(\frac{8}{40 - 23}\right) \times 10$$

$$\Rightarrow \text{Mode} = 40 + \left(\frac{8}{17}\right) \times 10$$

$$\Rightarrow \text{Mode} = 40 + 4.71$$

$$\therefore \text{Mode} = 44.7 \text{ cars}$$

17 ) 80 ( 4.705
68
120
119
100
85
15

$\frac{80}{17} = 4.705 = 4.71$

2.The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data

| Monthly consumption (in units) | 65-85 | 85-105 | 105-125 | 125-145 | 145-165 | 165-185 | 185-205 |
|---|---|---|---|---|---|---|---|
| Number of consumers | 4 | 5 | 13 | 20 | 14 | 8 | 4 |

Solution:

| Monthly consumption (in units) | Number of consumers ($f_i$) | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_i u_i$ | Cumulative frequency (c f) |
|---|---|---|---|---|---|
| 65-85 | 4 | 75 | −3 | −12 | 4 |
| 85-105 | 5 | 95 | −2 | −10 | 4 + 5 = 9 |
| 105-125 | 1[illegible] ← $f_0$ | 115 | −1 | −13 | 9 + 13= 22 ( c f ) |
| 125-145 f → | 2[illegible] ← $f_1$ | 135 ($a$) | 0 | 0 | 22 + 20 = 42 |
| 145-165 | 1 ← $f_2$ | 155 | 1 | 14 | 42 + 14 = 56 |
| 165-185 | 8 | 175 | 2 | 16 | 56 + 8 = 64 |
| 185-205 | 4 | 195 | 3 | 12 | 64 + 4 = 68 |
| Total $\sum f_i = 68$ | | | $\sum f_i u_i = 7$ | | |

i) Mean

We know that, Class mark , $x_i = \frac{\text{Upper class limit+ Upper class limit}}{2}$

Class size, h = 20, Taking assumed mean, $a = 135$

From the table, we obtain $\sum f_I = 68$, $\sum f_i u_I = 7$

$$\text{Mean}, (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\Rightarrow \bar{x} = 135 + \left(\frac{7}{68}\right) \times 20 = 135 + \left(\frac{7}{68_{17}}\right) \times 20^{5}$$

17 ) 35 ( 2.05
34
100
85
15

$\frac{35}{17} = 2.05$

$$\Rightarrow \bar{x} = 135 + \frac{35}{17} = 135 + 2.05 = 137.05 \text{ units}$$

$$\therefore \bar{x} = 137.05 \text{ units.}$$

ii) Median From the table, it can be observed that $n = 68 \Rightarrow \frac{n}{2} = \frac{68}{2} = 34$

Cumulative frequency (c f) just greater than 34 is 42, belonging to class 125– 145.

Therefore, median class = 125– 145, Class size, h = 20 , Lower limit of the median class, $l = 125$

. Frequency of the median class, f = 20 , Cumulative frequency of class preceding median class, cf = 22

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h \Rightarrow \text{Median} = 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

$$\Rightarrow \text{Median} = 125 + \left(\frac{12}{20}\right) \times 20 = 125 + 12 = 137 \text{ units}$$

$$\therefore \text{Median} = 137 \text{ units}$$

iii) Mode

Solution: From the table, it can be observed that the maximum class frequency 20,

belonging to class interval 125– 145  Therefore, Modal class = 125– 145,  Class size, h =20

Lower limit of modal class, $l = 125$ ,  Frequency of modal class, $f_1 = 20$

Frequency of class preceding modal class, $f_0 = 13$ ,

Frequency of class succeeding the modal class, $f_2 = 14$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\text{Mode} = 125 + \left(\frac{20 - 13}{2\times20 - 13 - 14}\right) \times 20 \Rightarrow \text{Mode} = 125 + \left(\frac{7}{40 - 27}\right) \times 20$$

$$\Rightarrow \text{Mode} = 125 + \left(\frac{8}{13}\right) \times 20$$

$$\Rightarrow \text{Mode} = 125 + \left(\frac{140}{13}\right) \quad \Rightarrow \text{Mode} = 125 + 10.76$$

$$\therefore \text{Mode} = 135.76 \text{ units}$$

17 ) 140 ( 10.76
13
100
91
90
78
12

$\frac{140}{13} = 10.76$

3.The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is Rs.18. Find the missing frequency f.

| Daily protect allowance (in ₹`) | 11-13 | 13-15 | 15-17 | 17-19 | 19-21 | 21-23 | 23-25 |
|---|---|---|---|---|---|---|---|
| Number of children | 7 | 6 | 9 | 13 | f | 5 | 4 |

Solution: We know that, Class mark, $x_i = \frac{\text{Upper class limit+ Upper class limit}}{2}$

Class size, h = 2

Taking assumed mean, $a = 18$

| Daily protect allowance (in ₹) | Number of children($f_i$ ) | $x_i$ | $u_i = \frac{x_i - a}{h}$ | $f_i u_i$ |
|---|---|---|---|---|
| 11-13 | 7 | 12 | $-3$ | $-21$ |
| 13-15 | 6 | 14 | $-2$ | $-12$ |
| 15-17 | 9 | 16 | $-1$ | $-9$ |
| 17-19 | 13 | 18 ($a$) | 0 | 0 |
| 19-21 | f | 20 | 1 | f |
| 21-23 | 5 | 22 | 2 | 10 |
| 23-25 | 4 | 24 | 3 | 12 |
| Total | $\sum f_i = 44 + f$ | | $\sum f_i u_i = f - 20$ | |

Step deviation method: Given Mean = ₹18

From the table, we obtain $\sum f_i = 44 + f$ , $\sum f_i u_i = f - 20$,

$$\text{Mean}, (\bar{x}) = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

$$\Rightarrow \text{Mean} = 18 + \left(\frac{f-20}{44+f}\right) \times 2$$

$$\Rightarrow 18 = 18 + \left(\frac{f-20}{44+f}\right) \times 2$$

$$\Rightarrow 18 - 18 = \left(\frac{f-20}{44+f}\right) \times 2 \quad \Rightarrow 0 = \left(\frac{f-20}{44+f}\right) \times 2$$

$$\Rightarrow 0 = \frac{f-20}{44+f} \Rightarrow 0 \times (44 + f) = f - 20$$

$$\Rightarrow f - 20 = 0 \quad \Rightarrow \quad f = 20$$

$\therefore$ The missing frequency f = 20

4.If the median of the distribution given below is 28.5. Find the value of $x$ and $y$.

| C.I | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | Total |
|---|---|---|---|---|---|---|---|
| frequency | 5 | x | 20 | 15 | y | 5 | 60 |

Solution:

| C.I | frequency($f_i$ ) | Cumulative frequency (c f) |
|---|---|---|
| 0-10 | 5 | 5 |
| 10-20 | $x$ | $5 + x$ ( c f ) |
| 20-30 | 20 | $5 + x + 20 = 25 + x$ |
| 30-40 | 15 | $25 + x + 15 = 40 + x$ |
| 40-50 | $y$ | $40 + x + y$ |
| 50-60 | 5 | $40 + x + y + 5 = 45 + x + y$ |
| Total | $n = 45 + x + y$ | |

given $n = 60 \Rightarrow 60 = 45 + x + y \Rightarrow 60 = x + y = 60 - 45 = 15$

i.e., $x + y = 15 - - - - - - - - (i)$

Median of the data is given as 28.5 which lies in interval $20 - 30$.

Therefore, median class = 20 – 30 , Class size, h =10

Lower limit of the median class, $l = 20$ , Frequency of the median class, f = 20

Cumulative frequency of class preceding median class, $cf = 5 + x$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h \Longrightarrow 28.5 = 20 + \left(\frac{30 - (5 + x)}{20}\right) \times 10$$

$$\Rightarrow 28.5 - 20 = \left(\frac{30-5-x)}{2}\right) \Rightarrow 8.5 = \left(\frac{25-x}{2}\right)$$

$$\Rightarrow 8.5 \times 2 = 25 - x \Rightarrow 17 = 25 - x \Rightarrow x = 25 - 17 = 8$$

$x = 8$ now substitute $x = 8$ in eq (i) Then, $8 + y = 15 \Rightarrow y = 15 - 8 = 7$

The value of $x = 8$ and $y = 7$

**4 MARKS QUESTION ANSWERS**

1.Write the formula for mode for grouped data? Explain each term in it.?

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where $l$ = lower boundary of the modal class

$f_1$ = frequency of the modal class

$f_0$ = frequency of the class preceding the modal class

$f_2$ = frequency of the class succeeding the modal class

$h$ = size of the modal class

2.Write the formula for median for grouped data? Explain each term in it?

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

*where* $l$ = lower boundary of median class,

$n$ = number of observations,

$cf$ = cumulative frequency of class preceding the median class,

$f$ = frequency of median class,

$h$ = class size of median class

3.Write the formula for mean by assumed mean method? Explain each term in it?

$$\text{Mean}\,,(\bar{x}\,) \;=\; a + \left(\frac{\sum \mathrm{f_i d_i}}{\sum \mathrm{f_i}}\right)$$

$where\ a =$ assumed mean

$\mathrm{f_i} =$ frequency

$d_{\mathrm{i}} =$ deviation, $d_{\mathrm{i}} = x_i - a$,

$x_i =$ class mark, (or) mid value

4.Write the formula for mean by step deviation method? Explain each term in it?

$$\text{Mean}\,,(\bar{x}\,) = a + \left(\frac{\sum \mathrm{f_i u_i}}{\sum \mathrm{f_i}}\right) \times \mathrm{h}$$

$where\ a =$ assumed mean

$\mathrm{f_i} =$ frequency

unit interval $u_{\mathrm{i}} = \frac{d_{\mathrm{i}}}{\mathrm{h}} = \frac{x_i - a,_{\mathrm{i}}}{\mathrm{h}}$

$x_i =$ class mark,

$h =$ class size

# 14.PROBABILITY

**[13 Marks] [8 + 4 + 1 = 13 M]**

## LEVEL-1: RISING STAR

## 8 MARKS QUESTION ANSWERS

1.One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting,

i) a King of red colour    ii) a face card    iii) a Black face card.

iv) the Jack of Diamonds    v) a Spade    vi) The Queen of Hearts

vii) an ace of Red colour    viii) not a Black of Jack

Solution: Total number of cards from a well-shuffled deck = 52

No of spade cards = 13, No of heart cards = 13, No of diamond cards = 13 , No of club cards = 13

Total number of kings = 4, Total number of red colour kings = 2, Total number of queens = 4 ,

Total number of jacks = 4 ,No of face cards = 12, No of black face cards = 6

Total number of black jacks =2, Total number of not a black of jacks cards = 52 − 2=50.

i) a King of red colour

$$\text{Probability of getting a king of red colour} = \frac{\text{Number of red colour king}}{\text{total no of outcomes}} = \frac{2}{52} = \frac{\cancel{2}^{1}}{\cancel{52}_{26}} = \frac{1}{26}$$

ii) a face card

$$\text{Probability of getting a face card} = \frac{\text{Number of face cards}}{\text{total no of outcomes}} = \frac{12}{52} = \frac{\cancel{12}^{3}}{\cancel{52}_{13}} = \frac{3}{13}$$

iii) a Black face card.

$$\text{Probability of getting a black face card} = \frac{\text{Number of black face cards}}{\text{total no of outcomes}} = \frac{6}{52} = \frac{\cancel{6}^{3}}{\cancel{52}_{26}} = \frac{3}{26}$$

iv) the Jack of Diamonds

$$\text{Probability of getting the jack of diamonds} = \frac{\text{Number of jack of diamonds}}{\text{total no of outcomes}} = \frac{1}{52}$$

v) a Spade

$$\text{Probability of getting a spade card} = \frac{\text{Number of spade cards}}{\text{total no of outcomes}} = \frac{13}{52} = \frac{\cancel{13}^{1}}{\cancel{52}_{4}} = \frac{1}{4}$$

vi) The Queen of Hearts

$$\text{Probability of getting the queen of hearts} = \frac{\text{Number of queen of hearts}}{\text{total no of outcomes}} = \frac{1}{52}$$

vii) an ace of Red colour

$$\text{Probability of getting a ace of red colour} = \frac{\text{Number of red colour ace}}{\text{total no of outcomes}} = \frac{2}{52} = \frac{\overset{1}{\cancel{2}}}{\underset{26}{\cancel{52}}} = \frac{1}{26}$$

viii) not a Black of Jack

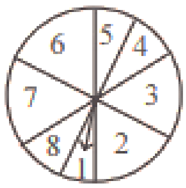
$$\text{Probability of getting the not a Black of Jack} = \frac{\text{total number of not a Black of Jack cards}}{\text{total no of outcomes}} = \frac{50}{52} = \frac{25}{26}$$

2.A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 see figure below. And these are equally likely outcomes.

What is the probability that it will point at?

i) '7' ?  ii) An Even Number

iii) A Number greater than '3' ?  iv) A Number less than 9?




(i) Probability of getting 7

The outcomes favourable to the event 'of getting 7' = 1

$$\text{Probability of getting 7} = = \frac{\text{favourable out comes}}{\text{total no of outcomes}} = \frac{1}{8}$$

(ii) Total no of even numbers = 2,4,6,8 = 4

$$\text{Probability of getting an even number} = \frac{\text{total no of even numbers}}{\text{total no of outcomes}} = \frac{4}{8} = \frac{1}{2}$$

(iii) Numbers greater than 3 are 4,5,6,7,8= 5

$$\text{Probability of getting Numbers greater than 3} = \frac{\text{no.of numbers greater than 3}}{\text{total no of outcomes}} = \frac{5}{8}$$

(iv) Numbers less than 9 are1,2 3,4,5,6,7,8 = 8

$$\text{Probability of getting a numbers less than 9} = \frac{\text{no.of numbers less than 9}}{\text{total no of outcomes}} = \frac{8}{8} = 1$$

3.A box contains 90 discs which are numbered from 1 to 90.

If one disc is drawn at random from the box, find the probability that it bears.

i) a two-digit number  ii) a perfect square number

iii) a number divisible by '5'.  iv) a perfect cube number

Solution: Total number of discs = 90

Total number of 2digit numbers between 1 to 90 = 81

Total number of perfect square numbers between1 to 90 are 1,4,9,16,25,36,49,64,81= 9

Total numbers that are divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90 = 18

Total number of a perfect cube numbers between1 to 90 are 1,8,27,64 = 4

i) a two-digit number

$$\text{Probability of getting a two-digit number} = \frac{\text{Total number of 2digit numbers between 1 to 90}}{\text{total no of outcomes}} = \frac{81}{90} = \frac{\overset{9}{\cancel{81}}}{\underset{10}{\cancel{90}}} = \frac{9}{10}$$

ii) a perfect square number

$$\text{Probability of getting a perfect square number} = \frac{\text{Total number of perfect square numbers between1 to 90}}{\text{total no of outcomes}} = \frac{9}{90} = \frac{\overset{1}{\cancel{9}}}{\underset{10}{\cancel{90}}} = \frac{1}{10}$$

iii) a number divisible by '5'.

$$\text{Probability of getting a number divisible by '5'} = \frac{\text{Total numbers that are divisible by 5 between1 to 90}}{\text{total no of outcomes}} = \frac{18}{90} = \frac{\overset{1}{\cancel{18}}}{\underset{5}{\cancel{90}}} = \frac{1}{5}$$

iv) a perfect cube number

$$\text{Probability of getting a perfect cube number} = \frac{\text{Total number of a perfect cube numbers between1 to 90}}{\text{total no of outcomes}} = \frac{4}{90} = \frac{\overset{2}{\cancel{4}}}{\underset{45}{\cancel{90}}} = \frac{2}{45}$$

4.A die is thrown once. Find the probability of getting

i) A prime number ii) a number greater than 4

iii) An odd number iv) Factors of '2'

Solution:

i) A prime number

Number of outcomes when you throw a die (1,2,3,4,5,6) = 6

Number of prime numbers on dice are 2,3 and 5 = 3

$$\text{Probability of getting a prime number} = \frac{\text{Number of prime numbers on dice}}{\text{total no of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

ii) a number greater than 4

Number of numbers greater than 4 are {5, 6} =2

$$\text{Probability of getting a number greater than 4} = \frac{\text{Number of numbers greater than 4}}{\text{total no of outcomes}} = \frac{2}{6} = \frac{1}{3}$$

iii) An odd number

Total number of odd numbers are 1,3 and 5 = 3

$$\text{Probability of getting an odd number} = \frac{\text{Number of odd numbers on dice}}{\text{total no of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

iv) Factors of '2'

Total number Factors of '2' are 1,2 = 2

$$\text{Probability of getting Factors of '2'} = \frac{\text{Number Of factors of '2'}}{\text{total no of outcomes}} = \frac{2}{6} = \frac{1}{3}$$

## 4 MARKS QUESTIONS ANSWERS

1.When a die is rolled once, find the probabilities of
i) getting a number between 3 and 6, now create 4 such type of questions.

i). When a die is rolled once, find the probabilities of
i) getting a number between 1 and 5?

ii). When a die is rolled once, find the probabilities of
i) getting a number between 2 and 6?

iii). When a die is rolled once, find the probabilities of
i) getting a number between 3 and 4?

iv). When a die is rolled once, find the probabilities of
i) getting a number between 1 and 4?

2. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the drawn card will be "the king"? Now create 4 such type of questions.

i). One card is drawn from a well-shuffled deck of 52 cards.
Calculate the probability that the drawn card will be "the ace"?

ii). One card is drawn from a well-shuffled deck of 52 cards.
Calculate the probability that the drawn card will be "the queen"?

iii). One card is drawn from a well-shuffled deck of 52 cards.
Calculate the probability that the drawn card will be "the jack"?

iv). One card is drawn from a well-shuffled deck of 52 cards.
Calculate the probability that the drawn card will be "the black king"?

3. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be green? Now create 4 such type of questions.

i). A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be red?

ii). A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be white?

iii). A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will not be green?

iv). A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be red and green?

## 1 MARK QUESTION ANSWERS

1.P(E) + P $(\bar{E})$ = ........... (C)

A) 0 B) – 1 C) 1 D) 1.5

2. P(E) = 0.05 what is the probability of "not E"? the probability of "not E" = 0.95

P (not E) = 1 – P (E) = 1 – 0.05 = 0.95

3.Which of the following cannot be the probability of an event? (B )

A) $\frac{2}{3}$ B)1.5 C) 15% D) 0.7

4.One card is drawn from a well shuffled deck of 52 cards. Calculate the probability that the card will

(i) be an ace (ii) not be an ace.

(i) be an ace

Probability of that the card will be an ace P (E) = $\frac{\text{Number of ace cards}}{\text{total no of outcomes}} = \frac{4}{52} = \frac{1}{13}$

(ii) not be an ace.

P (not be an ace) = 1 – P (E) = 1 – $\frac{1}{13} = \frac{13-1}{13} = \frac{12}{13}$

5.The sum of the probabilities of all the elementary events of an experiment is ........ ( B )

A )0 B) 1 C) 2 D) infinite

6. The probability of getting at least one Tail, when two coins are tossed is ...... ( A )

A) $\frac{3}{4}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{4}$

7.If a letter is taken at random from the letters in the word "MOBILE", what is the probability that it will be a "consonants" letter?

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{6}$ ( B)

8.The probability of getting a number less than 4 on the top when rolling a die is ( A )

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{5}{6}$ D) 0.75

9. A card is drawn at random from a well-shuffled deck of 52 cards. Calculate probability of a Red King?

A) $\frac{1}{13}$ B) $\frac{2}{13}$ C) $\frac{1}{26}$ D) $\frac{1}{52}$ ( C )

10.A bag contain 3 green marbles, 4 blue marbles and 2 orange marbles. What is the probability that a coloured marble drawn at random will not be an green marbles? ( B )

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{4}{9}$ D) $\frac{7}{9}$

11. What is the probability that a number is chosen at random from a two-digit number and that it is a multiple of 3?

A) $\frac{1}{3}$ B) $\frac{3}{10}$ C) $\frac{7}{25}$ D) $\frac{29}{100}$ ( A )

12. The probability of getting more than 5 on the top of a die is (C )

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{1}{6}$ D) $\frac{3}{5}$

13. The probability that a die rolled will result in a prime number is ( B )

A) $\frac{2}{3}$ B) $\frac{1}{2}$ C) 1 D) $\frac{1}{6}$

14. If a number is drawn at random from the 8 numbers 1, 2, 3, 4, 5, 6, 7, 8

what is the probability that it is a factors of '6'? ( A )

A) $\frac{1}{2}$ B) $\frac{5}{6}$ C) $\frac{3}{4}$ D) $\frac{7}{8}$

15. If 3 coins are tossed at the same time, the probability of not getting heads is ( D )

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{7}{8}$ D) $\frac{1}{8}$

These outcomes are: {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.

## LEVEL-2: SHINING STAR

### 8 MARKS QUESTION ANSWERS

1. Two dice are thrown at the same time.

i)Write all possible outcomes.

ii)What is the probability that sum of two numbers appearing on the top of the dice

a) 7 b) 16

iii)Find the probability of same number on both sides.

Solution:

When the first dice shows '1', the second dice could show any one of the numbers 1, 2, 3, 4, 5, 6.
The same is true when the first dice shows '2', '3', '4', '5' or '6'.
The possible outcomes of the experiment are listed in the table below;
the first number in each ordered pair is the number appearing on the first dice and the second number is that on the second dice.

i)All possible outcomes:

| | SECOND DICE | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| FIRST DICE | 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| | 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| | 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| | 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| | 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| | 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

ii) the probability that sum of two numbers appearing on the top of the dice a) 7

| | SECOND DICE | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| FIRST DICE | 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| | 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| | 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| | 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| | 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| | 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

a)The outcomes favourable to the event 'the sum of the two numbers appearing on the top of the dice is 7' denoted by E, are: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6,1) (see from the table )

i.e., the number of outcomes favourable to E = 5.

$$\text{So P ( E )} = \frac{\text{favourable out comes}}{\text{total no of outcomes}} = \frac{6}{36} = \frac{\cancel{6}^{1}}{\cancel{36}_{6}} = \frac{1}{6}$$

ii) b) As you can see from the possible outcomes table there is no outcome favourable to the event F 'the sum of two numbers is 16'.

$$\text{So, P (F)} = \frac{\text{favourable out comes}}{\text{total no of outcomes}} = \frac{0}{36} = 0$$

iii)The probability of same number on both dice:

| | SECOND DICE | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| FIRST DICE | 1 | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| | 2 | (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| | 3 | (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| | 4 | (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| | 5 | (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| | 6 | (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

The outcomes favourable to the event 'same number on both dice' denoted by G, are: (1, 1), (2, 2), (2, 2), (4, 4), (5, 5) , (6, 6) (see from the table )

$$\text{So P ( G )} = \frac{\text{favourable out comes}}{\text{total no of outcomes}} = \frac{6}{36} = \frac{\cancel{6}^{1}}{\cancel{36}_{6}} = \frac{1}{6}$$

2.A game consists of tossing a one-rupee coin '3' times and nothing its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and losses other- wise. Calculate the probability that Hanif will lose the game.

Solution: Total possible outcomes = [ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} = 8

| | | |
|---|---|---|
| H | H | H |
| H | H | T |
| H | T | H |
| H | T | T |
| T | H | H |
| T | H | T |
| T | T | H |
| T | T | T |

Number of possible outcomes to get three heads or three tails= 2

$$\text{Probability that Hanif will win the game} = \frac{\text{Number of possible outcomes}}{\text{total number of outcomes}} = \frac{2}{8} = \frac{\cancel{2}^{1}}{\cancel{8}_{4}} = \frac{1}{4}$$

$$\text{Probability that Hanif will lose the game} = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

$$\text{The probability that Hanif will lose the game} = \frac{3}{4}$$

3.A bag contains cards with numbers from 1 to 36 written on them. After mixing the cards well, a card is chosen at random from them and the probability that it is the following card is calculated.

i)An odd number ii) Prime number iii) Perfect squares iv) Multiples of 3 v) Only Even composite number

vi)The number is neither prime number nor a composite number. vii)Factors of 24

viii) Only an odd prime number

Solution: Total number of cards = 36

i)An odd number

Total number of odd numbers {1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35} = 18 (or) [$36 \div 2 = 18$]

Probability of getting an odd number $= \frac{\text{Total number of odd numbers}}{\text{Total number of cards}} = \frac{18}{36} = \frac{1}{2}$

ii) Prime number

Total number of prime numbers {2,3,5,7,11,13,17,19,23,29,31} = 11

Probability of getting a prime number $= \frac{\text{Total number of prime numbers}}{\text{Total number of cards}} = \frac{11}{36}$

iii) Perfect squares

Total number of perfect square numbers from 1 to 36 are {1,4,9,16,25,36} = 6

Probability of getting perfect square numbers $= \frac{\text{Total number of perfect square numbers}}{\text{Total number of cards}} = \frac{6}{36} = \frac{1}{6}$

iv)Multiples of 3

Total numbers that are multiples of 3 from 1 to 36 are {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36} = 12

(or) [$36 \div 3 = 12$]

Probability of getting multiples of 3 $= \frac{\text{Total number of multiples of 3}}{\text{Total number of cards}} = \frac{12}{36} = \frac{1}{3}$

v) Only Even composite number

Total number of Only even composite numbers from 1 to 36 are

{ 4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36 }= 17

Probability of getting Only even composite numbers $= \frac{\text{Total number of Only even composite numbers}}{\text{Total number of cards}} = \frac{17}{36}$

vi)The number is neither prime number nor a composite number.

Total number of the number is neither prime number nor a composite number from 1 to 36 are {1} = 1

Probability of getting the number is neither prime number nor a composite number

$$= \frac{\text{Total number of the numbers is neither prime number nor a composite number}}{\text{Total number of cards}} = \frac{1}{36}$$

vii)Factors of 24

Total number of factors of 24 are $\{1,2,3,4,6,8,12,24\} = 8$

Probability of getting factors of 24 $= \frac{\text{Total number of factors of 24}}{\text{Total number of cards}} = \frac{8}{36} = \frac{2}{9}$

viii) Only an odd prime number

Total no of odd prime numbers $\{3,5,7,11,13,17,19,23,29,31\} = 10$

Probability of getting odd prime numbers $= \frac{\text{Total no of odd prime numbers}}{\text{Total number of cards}} = \frac{10}{36} = \frac{5}{18}$

4.Five cards. the ten, Jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

i) What is the probability that the card is the queen

ii) If the queen is drawn and put aside, what is the probability that the second card picked up

is (a) an ace? (b) a queen?

Solution: Total number of cards {the ten, Jack, queen, king and ace of diamonds} = 5

number of queen cards = 1

i) the probability that the card is the queen:

probability of getting the card is the queen $= \frac{\text{number of queen cards}}{\text{total no of cards}} = \frac{1}{5}$

ii) If the queen is drawn and put aside, the probability that the second card picked up is (a) an ace

If the queen is drawn and put aside

Then total number of cards {the ten, Jack, king and ace of diamonds} =4

number of ace cards = 1

probability of getting the card is an ace $= \frac{\text{number of ace cards}}{\text{total no of cards}} = \frac{1}{4}$

(b) If the queen is drawn and put aside, the probability that the second card picked up is a queen?

If the queen is drawn and put aside

Then total number of cards {the ten, Jack, king and ace of diamonds} =4

number of queen cards =0

probability of getting the card is queen$= \frac{\text{number of queen cards}}{\text{total no of cards}} = \frac{0}{4} = 0$

## 4 MARKS QUESTION ANSWERS

1.Two dice are thrown at the same time. What is the probability that the sum of two numbers appearing on the top of the dice is '7'? Now create 4 such type of questions.

i)Two dice are thrown at the same time. What is the probability that the sum of two numbers appearing on the top of the dice is '8'?

ii) Two dice are thrown at the same time. What is the probability that the sum of two numbers appearing on the top of the dice is '6'?

iii) Two dice are thrown at the same time. What is the probability that the sum of two numbers appearing on the top of the dice is '5'?

iv)Two dice are thrown at the same time. What is the probability that the sum of two numbers appearing on the top of the dice is '0'?

2. A coin tossed 4 times. Find the probability of getting all the tails? Now create '4' such type of questions.

i)A coin tossed 4 times. Find the probability of getting all the heads?

ii) A coin tossed 4 times. Find the probability of getting two tails?

iii)A coin tossed 4 times. Find the probability of getting two heads?

iv) A coin tossed 4 times. Find the probability of getting one tail?

3.A bag contains 3 yellow balls, 6 green balls 4 red balls and 2 white balls. One ball is taken out of the bag at random. What is the probability that the ball takes out will be a red ball? Now create 4 such type of questions.

Solution: i) A bag contains 3 yellow balls, 6 green balls 4 red balls and 2 white balls. One ball is taken out of the bag at random. What is the probability that the ball takes out will be a yellow ball?

ii)A bag contains 3 yellow balls, 6 green balls 4 red balls and 2 white balls. One ball is taken out of the bag at random. What is the probability that the ball takes out will be a green ball?

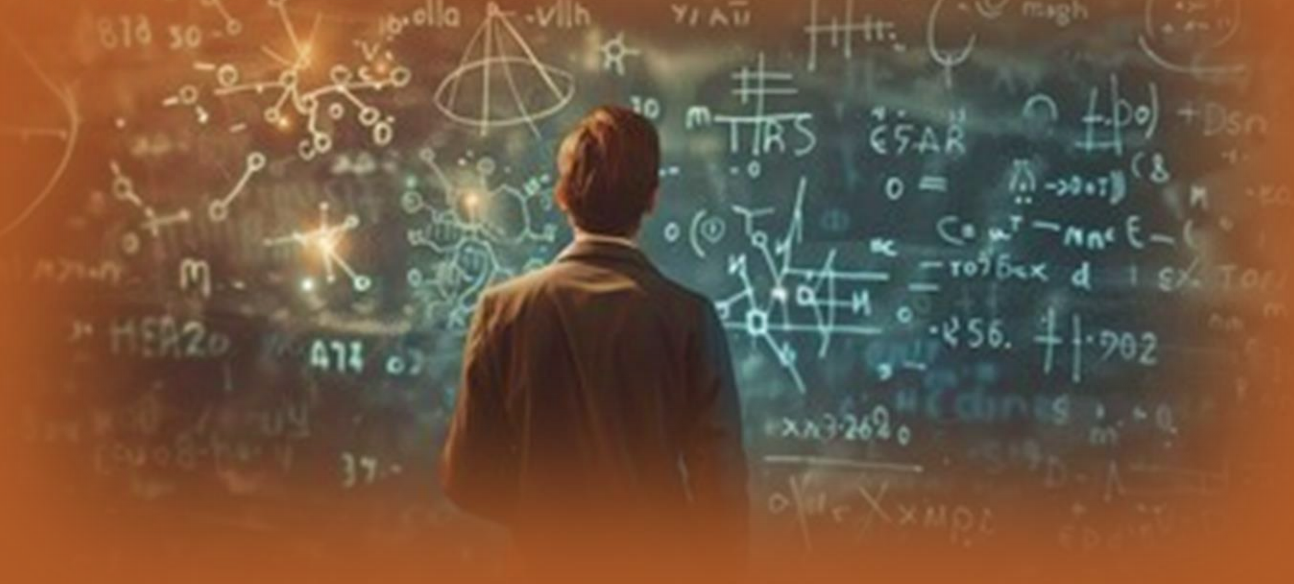
iii)A bag contains 3 yellow balls, 6 green balls 4 red balls and 2 white balls. One ball is taken out of the bag at random. What is the probability that the ball takes out will be a white ball?

iv) A bag contains 3 yellow balls, 6 green balls 4 red balls and 2 white balls. One ball is taken out of the bag at random. What is the probability that the ball takes out will be an orange ball?

*My dear students*

*These exams are not just a test of your knowledge but also of your dedication, perseverance, and hard work. Believe in yourself, stay focused, and give your best effort with confidence. Remember, success comes to those who prepare well and remain determined.*

## *Best wishes to students*



*By*
*B.J. Rammohan Rao,*
*Z.P. High School, Challapeta*
*S.A (Maths).*